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TECHNICAL NOTE 2505

ON THE ATTACHED CURVED SHOCK IN FRONT OF A SHARP-NOSED AXIALLY SYMMETRICAL BODY PLACED IN A UNIFORM STREAM

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SUMMARY

The flow behind the attached curved shock near the nose of an axially symmetrical body placed in a uniform stream is investigated by considering the perturbations from the initial Taylor-Maccoll conical solution. The first-order perturbation yields the ratio between the initial radii of curvature of the shock wave and the body. When higher-order perturbations are included, a regular shock wave near the nose leads to a body shape which has a logarithmic singularity at the nose. It seems, therefore, that, for a given regular body, the shock-wave shape probably has a singularity at the vertex, although the initial radius of curvature remains finite.

Numerical results are obtained for the first-order perturbation equations, covering the cases with initial semivertex angle $\theta_{s_0} = 10^{\circ}$, 20°, and 30°, each at five different Mach numbers ranging approximately from the minimum one for an attached conical shock to a value around 5. For each value of θ_{s_0}, there is a critical Mach number, very close to the minimum one for an attached conical shock, below which the ratio of curvatures becomes negative. This Mach number has been conjectured by Crocco in the two-dimensional case as the probable starting point for the detached shock wave. Its significance is discussed here on the basis of recent works by Thomas. The variation of the ratio of curvatures with Mach number is found to be of the same nature as that in the two-dimensional case, though the extent is much larger.

INTRODUCTION

The problem of the curved shock in two dimensions was first discussed by Crocco (reference 1) in 1937. Recently, papers by various authors (references 2 to 5) again indicate the current interest in the relation between the curvatures of the shock and the body. Lin and

Rubinov (reference 5) treated the flow behind curved shocks from a general viewpoint by expanding the hydrodynamical quantities behind the shock into Taylor series in Cartesian coordinates. They indicated that, by so doing, many problems in the axially symmetrical case could be solved in parallel with the two-dimensional ones. Nevertheless, when the curved shock is caused by a sharp-nosed body of revolution, singularity of the expansion is encountered at the nose and a different method should be applied.

This report represents an investigation of this particular problem, namely, the flow in the neighborhood of the sharp nose of a body of revolution, by means of a perturbation scheme. The difficulty arising out of an expansion in Cartesian coordinates is avoided by using polar coordinates. The relation between the initial curvatures of the shock and the body is thus obtained. On reaching the surface of the body, the first-order solution shows a logarithmic singularity at the initial semivertex angle, thereby apparently limiting the applicability only to concave bodies. It is suspected, however, that this difficulty actually arises from the asymptotic representation of the solution as used in this report and that the application of the result to convex bodies is permissible if the actual solution satisfies certain continuity conditions. It also appears from the asymptotic solution used in this report that, when high-order approximations are included, a regular shock wave would require the body behind it to have a singularity, presumably logarithmic in nature, at the nose. Conversely, this means that, if the body is representable by a regular function, the shock-wave shape might have a logarithmic singularity near the nose. The curvature of the shock would, however, stand in a finite ratio with the curvature of the body.

Numerical integrations have been carried out for the first-order perturbations for bodies with semivertex angles 10°, 20°, and 30° and a number of free-stream Mach numbers up to around 5. The variation of the ratio of the initial radii of curvature is found to be similar to that in the two-dimensional case as computed by Thomas (reference 3) or Munk and Prim (reference 4). A critical point beyond which the ratio of curvatures becomes negative likewise exists, such a point in the two-dimensional case being pointed out by Crocco (reference 1) as the probable limit of an actually attached shock wave.

The results of this report perhaps have significance as being the first step in clarifying the general problem of flow past an arbitrary body with an attached shock wave. On the practical side, its immediate application is to improve the accuracy of the usual method of characteristics by starting it by means of numerical computation at points away from the troublesome axis of symmetry. When the initial Taylor-Maccoll solution gives subsonic or partially subsonic flow behind the shock, the method of numerical integration by means of characteristics will fail to have any starting point. The present solution, if its validity is substantiated by experiment in this range of mixed subsonic and supersonic flows behind the shock, may then be used to determine approximately the sonic line, from which subsequent calculations may be made in the usual manner.

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SYMBOLS

θ, r	polar coordinates in a meridian plane
u, v, p, ρ	velocity components in r- and θ-directions, pressure, and density, respectively
c	"limit" velocity, a constant for isoenergetic flow
U, p^o, ρ^o, M^o	free-stream velocity, pressure, density, and Mach number, respectively
u_o, v_o, p_o, ρ_o	Taylor-Maccoll solution for initial vertex angle
$\overline{u}, \overline{v}, \overline{p}, \overline{\rho}$	perturbations from initial Taylor-Maccoll solution
u_1, v_1, p_1, ρ_1	factors depending on θ in first-order perturbations
u_n, v_n, p_n, ρ_n	factors depending on θ in nth-order perturbations
q_t, q_n	velocity components at the shock in tangential and normal directions, respectively
R	radius of curvature
ψ_w	angle between shock wave and uniform stream

F_1, F_2, F_3 G_1, G_2, G_3 H_1, H_2, H_3	coefficient functions in differential equations for first-order perturbations
ξ, η, ζ	nondimensional representation of u_1, v_1, and ρ_1, respectively
f_1, f_2, f_3 g_1, g_2, g_3 h_1, h_2, h_3	regular part of coefficient functions F_1, F_2, and so forth throughout interval of θ
$F_{k1}, F_{k2}, \ldots$	coefficient functions in differential equations for $(k + 1)$th-order perturbations
$f_{k1}, f_{k2}, \ldots$	regular part of coefficient functions F_{k1}, F_{k2}, and so forth throughout interval of θ

Subscripts:

s,w	quantities evaluated at body surface and shock wave, respectively
s_o, w_o	quantities evaluated at vertices of body surface and of shock wave, respectively

PERTURBATION EQUATION AND ITS BOUNDARY CONDITIONS

Consider a sharp-nosed body of revolution placed in a uniform stream in the direction of the axis of symmetry (fig. 1). In the neighborhood of the vertex, the shape of the body differs but slightly from that of a cone. One may therefore try to find a first-order perturbation to the well-known Taylor-Maccoll solution (reference 6), to be valid near the vertex. In view of the conical nature of the initial solution, it is logical to use spherical coordinates with the polar axis along the axis of the body. The surface of the body is represented by $\theta = \theta_s(r)$; the shock wave, by $\theta = \theta_w(r)$. The velocity components in the r- and θ-directions are denoted by u and v, respectively. The free-stream velocity is denoted by U, the pressure, by p^o, and the density, by ρ^o. The conditions immediately behind the curved shock are denoted by the subscript w and those on the surface of the body, by the subscript s. Needless to say, the flow behind the shock wave is still isoenergetic, though not irrotational.

With the introduction of polar coordinates, the equations of motion are:

$$u \frac{\partial u}{\partial r} + \frac{v}{r} \frac{\partial u}{\partial \theta} - \frac{v^2}{r} = -\frac{1}{\rho} \frac{\partial p}{\partial r} \tag{1}$$

$$u \frac{\partial v}{\partial r} + \frac{v}{r} \frac{\partial u}{\partial \theta} + \frac{uv}{r} = -\frac{1}{\rho} \frac{\partial p}{r\, \partial \theta} \tag{2}$$

The equation of continuity is

$$\frac{\partial}{\partial r} (\rho u) + \frac{\partial}{r\, \partial \theta} (\rho v) + 2 \frac{\rho u}{r} + \frac{\rho v}{r} \cot \theta = 0 \tag{3}$$

In addition there is Bernoulli's equation for isoenergetic flow,

$$\frac{2\gamma}{\gamma - 1} \frac{p}{\rho} = c^2 - u^2 - v^2$$

where c is the "limit" velocity, remaining constant throughout the field of flow, and γ, the ratio of specific heats. Equations (1) to (4) govern the four dependent variables p, ρ, u, and v. As is usually assumed, the solution starts with the Taylor-Maccoll solution for a cone of semivertex angle equal to the initial angle θ_{s_0} of the body. One tries to build upon it small perturbations to take care of the subsequent curvature. Consider then a perturbation scheme by writing

$$\left. \begin{aligned} u &= u_0(\theta) + \overline{u}(r,\theta) \\ v &= v_0(\theta) + \overline{v}(r,\theta) \\ \rho &= \rho_0(\theta) + \overline{\rho}(r,\theta) \\ p &= p_0(\theta) + \overline{p}(r,\theta) \end{aligned} \right\} \tag{5}$$

In equations (5) u_o, v_o, ρ_o, and p_o are the Taylor-Maccoll solution mention above. They satisfy the relations

$$\frac{du_o}{d\theta} - v_o = 0 \tag{6}$$

$$v_o\left(\frac{dv_o}{d\theta} + u_o\right) = -\frac{1}{\rho_o}\frac{dp_o}{d\theta} \tag{7}$$

$$\frac{d}{d\theta}\left(\rho_o v_o\right) + 2\rho_o u_o + \rho_o v_o \cot\theta = 0 \tag{8}$$

The barred quantities are the perturbations. Substituting equation (5) into equations (1) to (4), one obtains a set of equations involving u_o, v_o, and so forth, together with their perturbations. With the assumption of small perturbations, the quadratic terms of the perturbation quantities may be neglected and the following equations result:

$$\frac{1}{\gamma} u_o \frac{\partial\bar{u}}{\partial r} + v_o \frac{\partial\bar{u}}{r\,\partial\theta} - \frac{\bar{v}v_o}{r} - \frac{\gamma - 1}{\gamma} v_o \frac{\partial\bar{v}}{dr} + \frac{p_o}{\rho_o^2}\frac{\partial\bar{\rho}}{\partial r} = 0 \tag{9}$$

$$-\frac{\gamma - 1}{\gamma} u_o \frac{\partial\bar{u}}{r\,\partial\theta} + u_o\left(\frac{\partial\bar{v}}{\partial r} + \frac{\bar{v}}{r}\right) + \frac{1}{\gamma} v_o \frac{\partial\bar{v}}{r\,\partial\theta} + \frac{v_o}{r}\bar{u} + \frac{1}{\gamma}\bar{v}\frac{\partial v}{\gamma\,\partial\theta} -$$

$$\frac{\gamma - 1}{\gamma}\left(\frac{\partial u_o}{r\,\partial\theta} + \frac{u_o}{\rho_o}\frac{\partial\rho_o}{r\,\partial\theta}\right)\bar{u} - \frac{\gamma - 1}{\gamma}\frac{1}{\rho_o}\frac{\partial\rho_o}{r\,\partial\theta} v_o\bar{v} -$$

$$\frac{\gamma - 1}{\gamma} v_o \frac{\partial v_o}{r\,\partial\theta}\frac{\bar{\rho}}{\rho_o} - \frac{p_o}{\rho_o^2}\left(\frac{\bar{\rho}}{\rho_o}\frac{\partial\rho_o}{r\,\partial\theta} - \frac{\partial\bar{\rho}}{r\,\partial\theta}\right) = 0 \tag{10}$$

$$\rho_0\left(\frac{\partial \overline{u}}{\partial r} + \frac{2\overline{u}}{r}\right) + u_0\left(\frac{\partial \overline{\rho}}{\partial r} + \frac{2\overline{\rho}}{r}\right) + \rho_0 \frac{\partial \overline{v}}{r\ \partial \theta} + \overline{\rho}\left(\frac{\partial v_0}{r\ \partial \theta} + \frac{v_0}{r} \cot\ \theta\right) +$$

$$v_0 \frac{\partial \overline{\rho}}{r\ \partial \theta} + \overline{v}\left(\frac{\partial \rho_0}{r\ \partial \theta} + \frac{\rho_0}{r} \cot\ \theta\right) = 0 \tag{11}$$

One may try to find solutions of the form

$$\overline{u} = \beta_1(r)u_1(\theta)$$

$$\overline{v} = \beta_2(r)v_1(\theta)$$

$$\overline{\rho} = \beta_3(r)\rho_1(\theta)$$

where $\beta_1(r)$, $\beta_2(r)$, and $\beta_3(r)$ approach zero with r. This restriction is to ensure that the flow will approach the Taylor-Maccoll flow near the vertex, for all values of θ. After introducing this form of solution into equations (9) to (11), in general,

$$\beta_1(r) = \beta_2(r) = \beta_3(r) = r^n$$

where n is any positive number. For the immediate neighborhood of the vertex, it suffices to take the lowest value of n satisfying the boundary conditions. It then turns out, on assuming both the body and the shock shapes to have finite initial curvatures, that the only possibility to satisfy the boundary conditions is to take $n = 1$ (cf. equations (23) to (33)); that is,

$$\beta_1(r) = \beta_2(r) = \beta_3(r) = r \tag{12}$$

so that

$$\left.\begin{aligned} \overline{u} &= ru_1(\theta) \\ \overline{v} &= rv_1(\theta) \\ \overline{\rho} &= r\rho_1(\theta) \end{aligned}\right\} \tag{13}$$

The requirement on β_1, β_2, and β_3 for small values of r is automatically satisfied, and the solutions thus obtained should be valid asymptotic solutions for small values of r. Equations (9) to (11) for the perturbations then reduce to a set of ordinary linear differential equations:

$$\frac{1}{\gamma} u_o u_1 + v_o \frac{du_1}{d\theta} - \frac{2\gamma - 1}{\gamma} v_o v_1 + \frac{p_o}{\rho_o^2} \rho_1 = 0 \tag{14}$$

$$-\frac{\gamma - 1}{\gamma} u_o \frac{du_1}{d\theta} + u_1\left(\frac{1}{\gamma} v_o - \frac{\gamma - 1}{\gamma} \frac{u_o}{\rho_o} \frac{d\rho_o}{d\theta}\right) + \frac{1}{\gamma} v_o \frac{dv_1}{d\theta} +$$

$$v_1\left(2u_o + \frac{1}{\gamma} \frac{dv_o}{d\theta} - \frac{\gamma - 1}{\gamma} \frac{v_o}{\rho_o} \frac{d\rho_o}{d\theta}\right) + \frac{p_o}{\rho_o^2} \frac{d\rho_1}{d\theta} - \frac{p_o}{\rho_o^2} \frac{\rho_1}{\rho_o} \frac{d\rho_o}{d\theta} = 0 \tag{15}$$

$$3\rho_o u_1 + \rho_o \frac{dv_1}{d\theta} + v_1\left(\frac{d\rho_o}{d\theta} + \rho_o \cot \theta\right) + v_o \frac{d\rho_1}{d\theta} +$$

$$\rho_1\left(3u_o + \frac{dv_o}{d\theta} + v_o \cot \theta\right) = 0 \tag{16}$$

These equations may be put into the standard form of first-order linear differential equations for easier discussion, namely,

$$\frac{du_1}{d\theta} = F_1 u_1 + F_2 v_1 + F_3 \rho_1 \tag{17}$$

$$\frac{dv_1}{d\theta} = G_1u_1 + G_2v_1 + G_3\rho_1 \tag{18}$$

$$\frac{d\rho_1}{d\theta} = H_1u_1 + H_2v_1 + H_3\rho_1 \tag{19}$$

where the F's, G's, and H's are all functions of θ, containing the conical solutions u_o, v_o, and ρ_o. Explicitly,

$$\left.\begin{aligned} F_1 &= -\frac{1}{\gamma}\frac{u_o}{v_o} \\ F_2 &= \frac{2\gamma - 1}{\gamma} \\ F_3 &= -\frac{p_o}{\rho_o{}^2 v_o} \end{aligned}\right\} \tag{20}$$

$$\left.\begin{aligned} G_1 &= \frac{3\dfrac{p_o}{\rho_o} - \dfrac{1}{\gamma}v_o{}^2 + \dfrac{\gamma - 1}{\gamma}\dfrac{\rho_o'}{\rho_o}u_ov_o - \dfrac{\gamma - 1}{\gamma^2}u_o{}^2}{\dfrac{1}{\gamma}v_o{}^2 - \dfrac{p_o}{\rho_o}} \\ G_2 &= \frac{\dfrac{p_o}{\rho_o}\left(\dfrac{\rho_o'}{\rho_o} + \cot\theta\right) - v_o\left[2u_o + \dfrac{1}{\gamma}v_o' - \dfrac{\gamma - 1}{\gamma}\dfrac{\rho_o'}{\rho_o}v_o - \dfrac{(2\gamma - 1)(\gamma - 1)}{\gamma^2}u_o\right]}{\dfrac{1}{\gamma}v_o{}^2 - \dfrac{p_o}{\rho_o}} \\ G_3 &= \frac{p_o}{\rho_o{}^2}\,\frac{v_o' + v_o\left(\dfrac{\rho_o'}{\rho_o} + \cot\theta\right) + \left(\dfrac{2\gamma + 1}{\gamma}\right)u_o}{\dfrac{1}{\gamma}v_o{}^2 - \dfrac{p_o}{\rho_o}} \end{aligned}\right\} \tag{21}$$

$$\left.\begin{aligned}
H_1 &= \frac{-\frac{2}{\gamma} v_o - \frac{\gamma - 1}{\gamma} \frac{\rho_o'}{\rho_o} u_o + \frac{\gamma - 1}{\gamma^2} \frac{u_o^2}{v_o}}{\frac{1}{\rho_o}\left(\frac{1}{\gamma} v_o^2 - \frac{p_o}{\rho_o}\right)} \\
H_2 &= \frac{2u_o + \frac{1}{\gamma} v_o' - \left(\frac{\rho_o'}{\rho_o} + \frac{1}{\gamma} \cot \theta\right) v_o - \frac{(2\gamma - 1)(\gamma - 1)}{\gamma^2} u_o}{\frac{1}{\rho_o}\left(\frac{1}{\gamma} v_o^2 - \frac{p_o}{\rho_o}\right)} \\
H_3 &= \frac{-\frac{p_o}{\rho_o}\left(\frac{\rho_o'}{\rho_o} - \frac{\gamma - 1}{\gamma} \frac{u_o}{v_o}\right) - \frac{1}{\gamma} v_o\left(v_o' + 3u_o + v_o \cot \theta\right)}{\frac{1}{\gamma} v_o^2 - \frac{p_o}{\rho_o}}
\end{aligned}\right\} \tag{22}$$

As for the boundary conditions to be satisfied by the complete flow, they may be divided into those at the shock wave and that at the body surface. At the shock wave $\theta = \theta_w(r)$, the variables u, v, and ρ should be connected to the uniform stream by the well-known shock conditions. At the body surface, the resultant velocity should be tangent to the body. Consequently, for the perturbations u_1, v_1, and ρ_1, the following conditions must hold:

$$\left(ru_1, rv_1, r\rho_1\right)_{\theta=\theta_w} = \left(u_w - u_o, v_w - v_o, \rho_w - \rho_o\right)_{\theta=\theta_w} \tag{23}$$

$$\left(\frac{v_o + rv_1}{u_o + ru_1}\right)_{\theta=\theta_s} = \left(r \frac{d\theta_s}{dr}\right) \tag{24}$$

In condition (23), care must be taken to interpret correctly the exact meaning of the right-hand side. With reference to figure 2, the quantities u_w, u_o, and so forth are to be evaluated, strictly speaking, at point B, which is at an angle θ_w to the axis of symmetry and at which the shock wave is at an angle ψ_w with the uniform stream. It is now assumed that the shock-wave angle has no infinite curvature or higher-order derivatives and, therefore, is expressible as

$$\theta_w = \theta_{w_o} + r \frac{d\theta_w}{dr} + o(r) \tag{25}$$

Also,

$$\psi_w = \theta_w + \tan^{-1} \frac{r\, d\theta_w}{dr} \approx \theta_w + r \frac{d\theta_w}{dr} \tag{26}$$

For small values of r the shock conditions at B are therefore obtained by

$$u_w\Big|_{\theta=\theta_w} \approx u_w\Big|_{\theta=\theta_{w_o}} + 2r \frac{du_w}{d\psi} \frac{d\theta_w}{dr}\Big|_{\theta=\theta_{w_o}} \tag{27}$$

and similar expressions can be obtained for v_w and ρ_w. The argument for the determination of u_o, v_o, and ρ_o at $\theta = \theta_w$ runs along the same line. It may be noted that the conical solution is no longer regarded as only valid between the conical shock wave and the initial vertex angle of the body, but its validity has been extended analytically to the entire region with boundaries computed in reference 7.

Thus, from the values at point A lying on the line $\theta = \theta_{w_o}$ and having the same radius vector r as B, the values of u_o, and so forth at B may be evaluated. There follows,

$$u_o\Big|_{\theta=\theta_w} \approx u_o\Big|_{\theta=\theta_{w_o}} + r \frac{du_o}{d\theta} \frac{d\theta_w}{dr}\Big|_{\theta=\theta_{w_o}} \tag{28}$$

and similar expressions can be obtained for v_o and ρ_o. Remembering that in the conical solution at $\theta = \theta_{w_o}$,

$$u_w = u_o$$

$$v_w = v_o$$

$$\rho_w = \rho_o$$

one may reduce condition (23) to

$$\left(ru_1, rv_1, r\rho_1\right)_{\theta=\theta_w} = \left[r\left(2\,\frac{du_w}{d\psi} - \frac{du_o}{d\theta}\right)\frac{d\theta_w}{dr},\ .\ .\ .\right]_{\theta=\theta_{w_o}}$$

or

$$\left(u_1, v_1, \rho_1\right)_{\theta=\theta_w} = \left[\left(2\,\frac{du_w}{d\psi} - \frac{du_o}{d\theta}\right)\frac{d\theta_w}{dr}, \left(2\,\frac{dv_w}{d\psi} - \frac{dv_o}{d\theta}\right)\frac{d\theta_w}{dr}, \left(2\,\frac{d\rho_w}{d\psi} - \frac{d\rho_o}{d\theta}\right)\frac{d\theta_w}{dr}\right]_{\theta=\theta_{w_o}}$$

Since r is assumed to be a small parameter and the exceptional case of infinite curvature is excluded, the boundary conditions as stated above may actually be satisfied at $\theta = \theta_{w_o}$ instead of $\theta = \theta_w$ for the left-hand side. Hence the final form is

$$\left(u_1, v_1, \rho_1\right)_{\theta_{w_o}} = \left[\left(2\,\frac{du_w}{d\psi} - \frac{du_o}{d\theta}\right)\frac{d\theta_w}{dr}, \left(2\,\frac{dv_w}{d\psi} - \frac{dv_o}{d\theta}\right)\frac{d\theta_w}{dr}, \left(2\,\frac{d\rho_w}{d\psi} - \frac{d\rho_o}{d\theta}\right)\frac{d\theta_w}{dr}\right]_{\theta=\theta_{w_o}} \tag{29}$$

Condition (24) at the body surface may likewise be put into a more explicit form. Assuming now that near the vertex the body shape may be written as

$$\theta_s = \theta_{s_o} + r\,\frac{d\theta_s}{dr} + \text{Higher-order terms} \tag{30}$$

then

$$v_o\Big|_{\theta=\theta_s} \approx v_o\Big|_{\theta=\theta_{s_o}} + r\,\frac{dv_o}{d\theta}\,\frac{d\theta_s}{dr}\Big|_{\theta=\theta_{s_o}} = r\,\frac{dv_o}{d\theta}\,\frac{d\theta_s}{dr}\Big|_{\theta=\theta_{s_o}} \tag{31}$$

since $v_o\big|_{\theta=\theta_{s_o}} = 0$. As a result

$$\left(\frac{rv_1}{u_o + ru_1}\right)_{\theta=\theta_s} = \left[r\left(1 - \frac{1}{u_o}\frac{dv_o}{d\theta}\right)\frac{d\theta_s}{dr}\right]_{\theta=\theta_s} \tag{32}$$

By omitting the higher-order terms, the condition at the body surface finally reduces to

$$\left(\frac{v_1}{u_o}\right)_{\theta=\theta_{s_o}} = \left(1 - \frac{1}{u_o}\frac{dv_o}{d\theta}\right)\frac{d\theta_s}{dr}\bigg|_{\theta=\theta_{s_o}} \tag{33}$$

Regarding the problem now as an initial-value problem, one sees that from equation (29) the initial values of the variables are proportional to $\left.\frac{d\theta_w}{dr}\right|_{\theta=\theta_{w_o}}$, and the final value of v_1/u_o reached at the body as given by equation (33) is proportional to $\left.\frac{d\theta_s}{dr}\right|_{\theta=\theta_{s_o}}$. It therefore may be concluded that, because of the linearity of equations (17) to (19), the quantities $\left.\frac{d\theta_s}{dr}\right|_{\theta=\theta_{s_o}}$ and $\left.\frac{d\theta_w}{dr}\right|_{\theta=\theta_{w_o}}$ are simply proportional to each other. Restricted to the neighborhood of the nose, the first derivative $d\theta/dr$ in fact may be interpreted as the curvature. For, from elementary calculus,

$$\frac{1}{R} = \frac{d\psi}{ds} = \frac{2\frac{d\theta}{dr} + r\frac{d^2\theta}{dr^2} + r^2\left(\frac{d\theta}{dr}\right)^3}{\left[1 + r^2\left(\frac{d\theta}{dr}\right)^2\right]^{3/2}} \longrightarrow 2\frac{d\theta}{dr}$$

as $r \longrightarrow 0$.

The right-hand side of equation (29) involves derivatives of the oblique shock conditions and the conical solution. The explicit expressions can be easily obtained. Resolving the velocity behind shock into tangential and normal components with respect to the shock (fig. 3) it may be readily verified that, as $r \longrightarrow 0$, the following are true:

$$\frac{du_w}{d\psi} = \frac{dq_t}{d\psi} - \frac{1}{2} v_w \tag{34}$$

$$\frac{dv_w}{d\psi} = -\frac{dq_n}{d\psi} + \frac{1}{2} u_w \tag{35}$$

where $dq_t/d\psi$ and $dq_n/d\psi$ may be derived from the standard shock relations as

$$\frac{dq_t}{d\psi} = -U \sin \psi \tag{36}$$

$$\frac{dq_n}{d\psi} = U \cos \psi \left[\frac{2(\gamma - 1)}{\gamma + 1} - \frac{\rho^o}{\rho_w}\right] \tag{37}$$

The variation of density with shock angle is simply

$$\frac{1}{\rho^o} \frac{d\rho_w}{d\psi} = \left(\frac{\rho_w}{\rho^o}\right)^2 \frac{4 \cot \psi}{(\gamma + 1)\left(M^o\right)^2 \sin^2\psi} \tag{38}$$

The conical solution is tabulated in detail in reference 7. Maccoll (reference 8) has expanded the solution near the cone surface $\theta = \theta_{s_o}$ as

$$\begin{aligned}\frac{u_o}{u_{s_o}} = 1 - \left(\theta - \theta_{s_o}\right)^2 + \frac{\cot \theta_{s_o}}{3}\left(\theta - \theta_{s_o}\right)^3 - \\ \left[\cot^2\theta_{s_o} + \frac{8}{3(\gamma - 1)} \frac{\left(\frac{u_{s_o}}{c}\right)^2}{1 - \frac{\left(u_{s_o}\right)^2}{c^2}}\right] \frac{\left(\theta - \theta_{s_o}\right)^4}{4} + \\ \left[\cot^3\theta_{s_o} + \frac{0.5833 + \frac{87 - 7\gamma}{12(\gamma - 1)} \frac{\left(u_{s_o}\right)^2}{c^2}}{1 - \frac{\left(u_{s_o}\right)^2}{c^2}} \cot \theta_{s_o}\right] \frac{\left(\theta - \theta_{s_o}\right)^5}{5} + \ldots\end{aligned} \tag{39}$$

where u_{s_o} stands for the value of u_o at $\theta = \theta_{s_o}$. The series forms for v_o and ρ_o may be derived by substituting equation (39) into the differential equations. Also known is the fact that the conical solution is analytic for a range of θ larger than the closed interval $\theta_{w_o} \geqq \theta \geqq \theta_{s_o}$. This knowledge is important because in formulating boundary conditions (29) and (32) the analyticity of the conical solution has been used. In other words, in taking the derivatives $du_o/d\theta$, and so forth, both the shock wave and the conical body surface are considered to be absent and the conical solution is extended beyond the range $\theta_{w_o} \geqq \theta \geqq \theta_{s_o}$.

INTEGRATION OF PERTURBATION EQUATIONS

In view of the proportionality of $\left.\frac{d\theta_s}{dr}\right|_{\theta=\theta_{s_o}}$ and $\left.\frac{d\theta_w}{dr}\right|_{\theta=\theta_{w_o}}$ concluded from the formulation of the boundary conditions, equations (17) to (19) are to be put into nondimensional form by using the initial radius of curvature R_w of the shock wave as the characteristic length, the "limit" velocity c as the characteristic velocity, and the free-stream density ρ^o as the characteristic density. Let

$$\xi = \frac{2R_w u_1}{c}$$

$$\eta = \frac{2R_w v_1}{c}$$

$$\zeta = \frac{2R_w \rho_1}{\rho^o}$$

Then, equations (17) to (19) may be rewritten as

$$\xi' = \frac{f_1}{\theta - \theta_{s_o}} \xi + f_2\eta + \frac{f_3}{\theta - \theta_{s_o}} \zeta \tag{40}$$

$$\eta' = g_1\xi + g_2\eta + g_3\zeta \tag{41}$$

$$\zeta' = \frac{h_1}{\theta - \theta_{s_o}}\,\xi + h_2\eta + \frac{h_3}{\theta - \theta_{s_o}}\,\zeta \tag{42}$$

The functions f, g, and h are easily identified by comparing with equations (20) to (22). The results are:

$$\left.\begin{array}{lll} f_1 = \left(\theta - \theta_{s_o}\right)F_1 & g_1 = G_1 & h_1 = \frac{c}{\rho^o}\left(\theta - \theta_{s_o}\right)H_1 \\ f_2 = F_2 & g_2 = G_2 & h_2 = \frac{c}{\rho^o} H_2 \\ f_3 = \frac{\rho^o}{c}\left(\theta - \theta_{s_o}\right)F_3 & g_3 = \frac{\rho^o}{c} G_3 & h_3 = \left(\theta - \theta_{s_o}\right)H_3 \end{array}\right\} \tag{43}$$

A factor $\left(\theta - \theta_{s_o}\right)$ is here multiplied by F_1, F_3, H_1, and H_3 because it is recognized from equations (20) and (22) that each of these functions has a pole at $\theta = \theta_{s_o}$, where v_o varies as $\theta - \theta_{s_o}$. The functions f_1, and so forth are made regular for easier discussion.

The boundary conditions now become:

$$\left.\begin{array}{l} \xi = 2\left.\frac{d\left(u_w/c\right)}{d\psi}\right|_{\psi=\theta_{w_o}} - \left.\frac{d\left(u_o/c\right)}{d\theta}\right|_{\theta=\theta_{w_o}} \\ \eta = 2\left.\frac{d\left(v_w/c\right)}{d\psi}\right|_{\psi=\theta_{w_o}} - \left.\frac{d\left(v_o/c\right)}{d\theta}\right|_{\theta=\theta_{w_o}} \\ \zeta = 2\left.\frac{d\left(\rho_w/\rho^o\right)}{d\psi}\right|_{\psi=\theta_{w_o}} - \left.\frac{d\left(\rho_o/\rho^o\right)}{d\theta}\right|_{\theta=\theta_{w_o}} \end{array}\right\} \text{at} \quad \theta = \theta_{w_o} \tag{44}$$

$$\frac{\eta}{u_{s_0}/c} = \left(1 - \frac{1}{u_{s_0}/c} d \frac{v_{s_0}/c}{d\theta}\right)\frac{R_{w_0}}{R_{s_0}}$$

$$= 3 \frac{R_{w_0}}{R_{s_0}} \qquad \text{at} \quad \theta = \theta_{s_0} \tag{45}$$

since

$$\frac{\dfrac{d\left(v_{s_0}/c\right)}{d\theta}}{u_{s_0}/c} = -2$$

by equations (6) and (39). Here R_{s_0} is the initial radius of curvature of the body surface. Since the functions f, g, and h are known only in the form of numerical data such as those presented in reference 7, integration of equations (40) to (42) generally can only be done by numerical process. With given θ_{s_0} and M^0, one starts from the initial points represented by equations (44) and integrates stepwise until $\theta = \theta_{s_0}$ is reached. The value of η then bears out the ratio of the radii of curvature by equation (45).

The appearance of poles at $\theta = \theta_{s_0}$ in some of the coefficients of ξ and ζ in equations (40) to (42) indicates that singularities are to be expected in the solutions. As is well-known from the theory of differential equations (see, e.g., reference 9) the singularity here is in fact a "regular" one. If the solutions are assumed to be of the form

$$\xi = \left(\theta - \theta_{s_0}\right)^{\alpha} P(\theta)$$

$$\eta = \left(\theta - \theta_{s_0}\right)^{\alpha} Q(\theta)$$

$$\zeta = \left(\theta - \theta_{s_0}\right)^{\alpha} R(\theta)$$

where P, Q, and R are analytic at $\theta = \theta_{s_0}$, the exponent α may be determined from the indicial equation of equations (40) to (42):

$$\begin{vmatrix} f_{10} - \alpha & 0 & f_{30} \\ 0 & -\alpha & 0 \\ h_{10} & 0 & h_{30} - \alpha \end{vmatrix} = 0 \tag{46}$$

in which f_{10}, f_{30}, h_{10}, and h_{30} are the values of the functions f_1, f_3, h_1, and h_3, respectively, at $\theta = \theta_{s_o}$. Hence

$$\alpha\left[\alpha^2 - \alpha\left(f_{10} + h_{30}\right) + f_{10}h_{30} - f_{30}h_{10}\right] = 0 \tag{47}$$

It is easy to verify that, based on equation (39),

$$\left.\begin{aligned}
f_{10} &= \frac{1}{2\gamma} \\
f_{30} &= \frac{p_{s_o}}{2\rho_{s_o}{}^2 u_{s_o}} \\
h_{10} &= \frac{\gamma - 1}{2\gamma^2} \frac{\rho_{s_o}{}^2 u_{s_o}}{p_{s_o}} \\
h_{30} &= \frac{\gamma - 1}{2\gamma}
\end{aligned}\right\} \tag{48}$$

By substitution of equation (48) into the indicial equation (46), the latter becomes

$$\alpha^2\left(\alpha - \frac{1}{2}\right) = 0$$

with roots 0, 0, and 1/2. Consequently, the solutions near the singularity are of the form

$$\left.\begin{aligned}
\xi &= \sum_{n=0}^{\infty} a_{\xi n}\left(\theta - \theta_{s_0}\right)^n + \sum_{n=0}^{\infty} b_{\xi n}\left(\theta - \theta_{s_0}\right)^{n+\frac{1}{2}} + \\
&\qquad \log\left(\theta - \theta_{s_0}\right)\sum_{n=0}^{\infty} c_{\xi n}\left(\theta - \theta_{s_0}\right)^n \\
\eta &= \sum_{n=0}^{\infty} a_{\eta n}\left(\theta - \theta_{s_0}\right)^n + \sum_{n=0}^{\infty} b_{\eta n}\left(\theta - \theta_{s_0}\right)^{n+\frac{3}{2}} + \\
&\qquad \log\left(\theta - \theta_{s_0}\right)\sum_{n=0}^{\infty} c_{\eta n}\left(\theta - \theta_{s_0}\right)^{n+1} \\
\zeta &= \sum_{n=0}^{\infty} a_{\zeta n}\left(\theta - \theta_{s_0}\right)^n + \sum_{n=0}^{\infty} b_{\zeta n}\left(\theta - \theta_{s_0}\right)^{n+\frac{1}{2}} + \\
&\qquad \log\left(\theta - \theta_{s_0}\right)\sum_{n=0}^{\infty} c_{\zeta n}\left(\theta - \theta_{s_0}\right)^n
\end{aligned}\right\} \tag{49}$$

It may be noted that η is one degree higher in $\theta - \theta_{s_0}$ in the series with coefficients b_n and c_n, because of the nature of equation (41). This fact is fortunate because the logarithmic infinity at $\theta = \theta_{s_0}$ in the solution of η is thus eliminated, leaving a finite value for the ratio of the radii of curvature. The logarithmic terms in ξ and ζ are more troublesome. Although the actual perturbations are given by ru_1 and $r\rho_1$ (or $r\xi$ and $r\zeta$), at the point $\theta = \theta_{s_0}$ the perturbations are small only if $r \log_e\left(\theta - \theta_{s_0}\right) \longrightarrow 0$. If the body is concave, that is, $\theta \geqq \theta_{s_0}$, and is assumed to have a finite initial curvature, there is obtained

$$r_s = \frac{1}{\left(\dfrac{d\theta_s}{dr}\right)_{\theta=\theta_{s_0}}}\left(\theta - \theta_{s_0}\right) + \text{Higher-order terms} \tag{50}$$

Evaluated at the body surface, the combinations ru_1 and $r\rho_1$ obviously go to zero. The formulation of the boundary condition (45) is still valid, and one obtains a finite ratio R_{w_0}/R_{s_0} for each value of θ_{s_0} and free-stream Mach number M^0. On the other hand, if the body is convex, the region of flow involves both $\theta - \theta_{s_0} > 0$ and $\theta - \theta_{s_0} < 0$. The assumption of small perturbation breaks down at the point $\theta = \theta_{s_0}$ and the procedure adopted above requires further examination. A discussion of this point will be taken up in the section "Numerical Results and Discussion." Consider for the moment, then, only bodies concave near the vertex.

Of all the coefficients in the series solutions (49), only three may be chosen to fit the boundary conditions. In order to obtain the recurrence formulas for the rest, expand first

$$\left.\begin{aligned} f_1 &= \sum_{n=0}^{\infty} f_{1n}\left(\theta - \theta_{s_0}\right)^n \\ g_1 &= \sum_{n=0}^{\infty} g_{1n}\left(\theta - \theta_{s_0}\right)^n \\ h_1 &= \sum_{n=0}^{\infty} h_{1n}\left(\theta - \theta_{s_0}\right)^n \end{aligned}\right\} \tag{51}$$

and so on. By substituting into equations (40) to (42), the following equations are derived:

$$\left.\begin{aligned} & f_{10}c_{\xi 0} + f_{30}c_{\zeta 0} = 0 \\ & f_{10}c_{\xi 1} + f_{11}c_{\xi 0} + f_{30}c_{\zeta 1} + f_{31}c_{\zeta 0} - c_{\xi 1} = 0 \\ & \sum_{l+m=n}\left(f_{1l}c_{\xi m} + f_{3l}c_{\zeta m}\right) - nc_{\xi n} + f_2 c_{\eta(n-2)} = 0 \\ & \qquad\qquad (n = 2, 3, 4, \ldots) \end{aligned}\right\} \tag{52}$$

$$\left.\begin{aligned}
&g_{10}c_{\xi 0} + g_{30}c_{\zeta 0} - c_{\eta 0} = 0 \\
&\sum_{l+m=n} g_{1l}c_{\xi m} + g_{3l}c_{\zeta m} + \sum_{l+m=n-1} g_{2l}c_{\eta m} - (n + 1)c_{\eta n} = 0 \\
&\qquad\qquad (n = 1, 2, 3, \ldots)
\end{aligned}\right\} \tag{53}$$

$$\left.\begin{aligned}
&h_{10}c_{\xi 0} + h_{30}c_{\zeta 0} = 0 \\
&h_{10}c_{\xi 1} + h_{11}c_{\xi 0} + h_{30}c_{\zeta 1} + h_{31}c_{\zeta 0} - c_{\zeta 1} = 0 \\
&\sum_{l+m=n} \left(h_{1l}c_{\xi m} + h_{3l}c_{\zeta m}\right) - nc_{\zeta n} + \sum_{l+m=n-2} h_{2l}c_{\eta m} = 0 \\
&\qquad\qquad (n = 2, 3, 4, \ldots)
\end{aligned}\right\} \tag{54}$$

$$\left.\begin{aligned}
&f_{10}b_{\xi 0} + f_{30}b_{\zeta 0} - \frac{1}{2} b_{\xi 0} = 0 \\
&f_{10}b_{\xi 1} + f_{11}b_{\xi 0} + f_{30}b_{\zeta 1} + f_{31}b_{\zeta 0} - \frac{3}{2} b_{\xi 1} = 0 \\
&\sum_{l+m=n} \left(f_{1l}b_{\xi m} + f_{3l}b_{\zeta m}\right) - \left(n + \frac{1}{2}\right)b_{\xi n} + f_2 b_{\eta(n-2)} = 0 \\
&\qquad\qquad (n = 2, 3, 4, \ldots)
\end{aligned}\right\} \tag{55}$$

$$\left.\begin{aligned}
&g_{10}b_{\xi 0} + g_{30}b_{\xi 0} - \frac{3}{2}\, b_{\eta 0} = 0 \\
&g_{10}b_{\xi 1} + g_{11}b_{\xi 0} + g_{30}b_{\zeta 1} + g_{31}b_{\zeta 0} - \frac{5}{2}\, b_{\eta 1} + g_{20}b_{\eta 0} = 0 \\
&\sum_{l+m=n}\left(g_{1l}b_{\xi m} + g_{3l}b_{\zeta m}\right) - \left(n + \frac{3}{2}\right)b_{\eta n} + \sum_{l+m=n-1} g_{2l}b_{\eta m} = 0 \\
&\qquad\qquad (n = 2, 3, 4, \ldots)
\end{aligned}\right\} \quad (56)$$

$$\left.\begin{aligned}
&h_{10}b_{\xi 0} + h_{30}b_{\zeta 0} - \frac{1}{2}\, b_{\zeta 0} = 0 \\
&h_{10}b_{\xi 1} + h_{11}b_{\xi 0} + h_{30}b_{\zeta 1} + h_{31}b_{\zeta 0} - \frac{3}{2}\, b_{\zeta 1} = 0 \\
&\sum_{l+m=n}\left(h_{1l}b_{\xi m} + h_{3l}b_{\zeta m}\right) - b_{\zeta n}\left(n + \frac{1}{2}\right) + \sum_{l+m=n-2} h_{2l}b_{\eta m} = 0 \\
&\qquad\qquad (n = 2, 3, 4, \ldots)
\end{aligned}\right\} \quad (57)$$

$$\left.\begin{aligned}
&-c_{\xi 0} + f_{10}a_{\xi 0} + f_{30}a_{\zeta 0} = 0 \\
&\sum_{l+m=n}\left(f_{1l}a_{\xi m} + f_{3l}a_{\zeta m}\right) - na_{\xi n} - c_{\xi n} + f_{2}a_{\eta(n-1)} = 0 \\
&\qquad\qquad (n = 1, 2, 3, \ldots)
\end{aligned}\right\} \quad (58)$$

$$\sum_{l+m=n}\left(g_{1l}a_{\xi m} + g_{3l}a_{\zeta m} + g_{2l}a_{\eta m}\right) - c_{\eta n} - (n + 1)a_{\eta(n+1)} = 0$$

$$(n = 0, 1, 2, 3, \ldots) \qquad (59)$$

$$
\left.
\begin{aligned}
& h_{10}a_{\xi 0} + h_{30}a_{\zeta 0} - c_{\zeta 0} = 0 \\
& \sum_{l+m=n+1} \left(h_{1l}a_{\xi m} + h_{3l}a_{\zeta m} \right) - c_{\zeta(n+1)} + \\
& \qquad \sum_{l+m=n} h_{2l}a_{\zeta m} - a_{\zeta(n+1)}(n + 1) = 0 \\
& \qquad\qquad\qquad (n = 0, 1, 2, \ldots)
\end{aligned}
\right\} \tag{60}
$$

In getting nearer to $\theta = \theta_{s_0}$ the method of numerical integration may become inconvenient. The series forms are to take over from there on to indicate the behavior of the various quantities. The first three coefficients in the expansions (51) are given in appendix A.

In spite of the fact that logarithmic singularities occur in both ξ and ζ near the vertex, the omission of quadratic terms of the perturbations in the derivation of equations (9) to (11) does not lead to inconsistency when the body is concave. For, it is clear that each quadratic term will be one degree higher in r in comparison with the linear terms. As in the discussion following equations (49), one may put r proportional to $\theta - \theta_{s_0}$ along the body. Then the same arguments may be used to justify the omission.

The logarithmic nature of the solution does lead to other complications. First, one would suspect that a regular shock curve does not lead to a regular body shape, and vice versa. It has been show, however, that the ratio of the initial curvatures is finite (cf. equation (45)). Singularities are revealed only when higher derivatives are investigated (see appendix B). Another complication is associated with convex bodies. The logarithmic singularity of the solution apparently prevents one from applying boundary conditions on the body, as pointed out above. This difficulty presumably comes from the inadequate knowledge of the mathematical nature of the solution and the improper method of representation. The representation may in fact be interpreted as an asymptotic one and is shown to lead to useful results only in a region bounded by a curve of the nature of equation (50). There is reason to suspect, however, that the ratio of curvature calculated for concave bodies also holds for the convex case. Mathematically speaking, the asymptotic representation of a function, as has been adopted in this report, is known to exhibit rather frequently singularities which are absent in the function itself. For extensions of the present results to convex bodies, it is only necessary that the quantity $\partial^2 v/\partial r\, \partial\theta$ have a unique value for $\theta = \theta_{s_o}$ at the vertex in the exact solution. For, the value of $\partial^2 v/\partial r\, \partial\theta$ evaluated by following a path along the body surface would be proportional to the curvature of the body, while the present method of calculation gives a valid result if the path satisfies the restriction (50), say. If these are the same, the above calculations hold with only a reversal of the signs of R_{w_o} and R_{s_o}, and the ratio would not be changed. Physically, one may also expect that a change of the body curvature one way or another would produce similar changes at the shock. To be sure, these arguments are not conclusive, and the application of the results to convex bodies must be taken with reserve. On the other hand, one should note that if $\partial^2 v/\partial r\, \partial\theta$ does not have a unique value for $r = 0$, $\theta = \theta_{s_o}$, the stepwise integration by the method of characteristics will also require careful examination. A thorough investigation of the mathematical nature of the solution is indeed very interesting and very much desired.

NUMERICAL RESULTS AND DISCUSSION

Numerical integrations have been carried out for the perturbation equations as outlined in the preceding section. Bodies with initial semivertex angles $\theta_{s_o} = 10^\circ$, 20°, and 30° are considered with the free-stream Mach number ranging approximately from the minimum one for attached conical shock wave to a value around 5. It is found that both ξ and ζ remain manageable practically up to the body surface. Since their values near the body surface are not needed for the determination of the ratio of the curvatures, the series forms (49) were not used. The quantity η approaches a finite value at the body surface and is easily determined in the stepwise integration. Table 1 gives the coefficient functions F, G, and H as well as the values of the variables during the integration of the various cases. In the computation, Kopal's tables (reference 7) have been used as the correct conical solution. The coefficient functions are computed to four places in most cases and in appropriate small steps. The final value of η at the body surface is of particular interest. After conversion to the ratio of the initial radii of curvature according to equation (45), the results are listed in table 2 and plotted as figure 4.

Variation of curvature ratio R_{w_o}/R_{s_o} with Mach number M for given values of θ_{s_o}.- It is seen that, for given values of θ_{s_o}, as the Mach number decreases from a fairly high value the ratio R_{w_o}/R_{s_o}

tends to increase until a maximum is reached. Further decrease of the Mach number causes the ratio to come down rapidly, and at least in one case $\left(\theta_{s_0} = 20^\circ,\ M = 1.216\right)$ the computation actually gives a negative value at a quite low Mach number. With smaller values of θ_{s_0}, the ratio exhibits a more violent change with Mach number in comparison with the cases of larger values of θ_{s_0}, though qualitatively the tendency is similar. As is well-known, the conical flow near the nose may be completely subsonic, completely supersonic, or a mixture of the two. The Mach numbers determining the different regimes for the present θ_s's are taken down from reference 7 and marked in figure 4 for comparison. At first one might surmise that perhaps the ratio of R_{w_0}/R_{s_0} reaches its maximum at the end of the supersonic regime, comes down in the mixed regime, and goes into negative values when flow becomes completely subsonic. This turns out not to be the case.

Zero point of ratio R_{w_0}/R_{s_0}.- The zero point of the ratio R_{w_0}/R_{s_0} lies very close to, though is not exactly coincident with, the critical Mach number below which a completely subsonic flow prevails. On the other hand, it does not seem justifiable to conclude too much in this respect. The vanishing of R_{w_0}/R_{s_0} means an infinite curvature of the shock wave, which may be recalled to be contradictory to the assumption in deriving the boundary conditions (cf. equation (25)). Consequently, the effects of higher-order terms will enter in deciding the radius of curvature. Indeed, a similar phenomenon has been found in the two-dimensional case and investigated to some extent by various authors. Crocco (reference 1) was the first to notice the appearance of a theoretical negative curvature ratio in two-dimensional shocks. He conjectured that detachment might start at this stage because of the unlikely physical picture. Guderley (reference 10) studied the behavior of flows qualitatively by examining the hodograph plane. For a straight wedge with a shoulder, he claimed that the shock would start with infinite, but not negative, curvature when the wedge angle lies beyond the Crocco point. The solution for a curved wedge was assumed to be similar in nature to that of the straight wedge with a shoulder. Recent works by Thomas (references 11 and 12) further indicate that as soon as a subsonic regime begins to appear behind the shock, the shock must exhibit a singular behavior, even though the body is of regular shape. The axially symmetrical case is even more complicated. As a matter of fact, the assumption of a regular shock-wave shape near the vertex is likely to be untrue for all Mach numbers, according to an investigation presented in appendix B. The present results in the subsonic range must therefore be interpreted with reserve. The zero point is seen to occur only when the flow behind the initial shock is entirely subsonic.

Comparison with two-dimensional case.- A comparison of the results in the supersonic regime with the corresponding ones in two-dimensional flow over a wedge may next be made. Thomas' results (reference 3) are converted into the notations adopted in this report and plotted as figure 5. The general tendency is seen to be similar. For larger Mach numbers the difference in the ratio of curvatures in the two cases becomes small. As the lowest Mach number for attached straight shock is higher in the two-dimensional case, deviation is large for the lower Mach numbers. The variation of the ratio of curvatures is also much less violent than in the axially symmetrical case. For instance, for $\theta_{s_o} = 10^o$, the ratio of curvatures reaches a maximum of 4.5 in the two-dimensional case but goes beyond 40 in the axially symmetrical one.

Experimental data have not been available to the authors for checking the theory. The greatest interest, besides checking the theory for its applicability to concave bodies, is, of course, the extension of the results to convex bodies. The behavior in the subsonic regime requires more theoretical study as well as a thorough experimental investigation, for which the technique is admittedly much more difficult as the delicate nature of transonic flow enters the picture. It is hoped that a comparison with experimental data may soon be made to evaluate the usefulness of the report.

On the report itself, the theoretical difficulty of the singular point at $\theta = \theta_{s_o}$ and the exact nature of the higher-order perturbations deserve further examination. If the first-order perturbation, as presented here, is found to agree well with experiments, more computation is needed for a conclusive knowledge of the variation of R_{w_o}/R_{s_o}. The curve for $\theta_{s_o} = 10^o$ in figure 4 can only be regarded as tentative because of the small number of computed points and the violent variation. At least one or two intermediate values of θ_{s_o} between 10^o and 20^o should be computed so that interpolation may become possible for practical purposes. The limiting case for $M^o \longrightarrow \infty$ is also of sufficient interest to be included in any subsequent computation.

Massachussetts Institute of Technology
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APPENDIX A

COEFFICIENTS IN EXPANSION (51)

The first three coefficients in expansion (51) are as follows:

$$f_{10} = \frac{1}{2\gamma}$$

$$f_{11} = \frac{1}{4\gamma} \cot \theta_s$$

$$f_{12} = -\frac{1}{2\gamma}\left[1 + \frac{1}{4} \cot^2\theta_{s_o} + \frac{4/3}{\gamma - 1} \frac{u_{s_o}{}^2/c^2}{1 - \left(u_{s_o}{}^2/c^2\right)}\right]$$

$$f_2 = \frac{2\gamma - 1}{\gamma}$$

$$f_{30} = \frac{\gamma - 1}{4\gamma} \frac{\rho^o}{\rho_{s_o}} \frac{1 - \left(u_{s_o}{}^2/c^2\right)}{u_{s_o}/c}$$

$$f_{31} = \frac{\gamma - 1}{8\gamma} \frac{1 - \left(u_{s_o}{}^2/c^2\right)}{u_{s_o}/c} \frac{\rho^o}{\rho_{s_o}} \cot \theta_{s_o}$$

$$f_{32} = -\frac{\gamma - 1}{4\gamma} \frac{1 - \left(u_{s_o}{}^2/c^2\right)}{u_{s_o}/c} \frac{\rho^o}{\rho_{s_o}}\left[\frac{1}{4} \cot^2\theta_{s_o} + \frac{2\gamma - (8/3)}{\gamma - 1} \frac{u_{s_o}{}^2/c^2}{1 - \left(u_{s_o}{}^2/c^2\right)}\right]$$

$$g_{10} = -3 + \frac{2}{\gamma} \frac{u_{s_o}{}^2/c^2}{1 - \left(u_{s_o}{}^2/c^2\right)}$$

$$g_{11} = 0$$

$$g_{12} = -\frac{2}{\gamma - 1} \frac{u_{s_o}{}^2/c^2}{1 - \left(u_{s_o}{}^2/c^2\right)}\left[10 - \frac{2}{\gamma} + 6\,\frac{\gamma - 1}{\gamma}\,\frac{u_{s_o}{}^2/c^2}{1 - \left(u_{s_o}{}^2/c^2\right)}\right]$$

$$g_{20} = -\cot\theta_{s_o}$$

$$g_{21} = 2\cot\theta_{s_o}\csc 2\theta_{s_o} + \frac{4}{\gamma(\gamma - 1)}\,\frac{u_{s_o}{}^2/c^2}{1 - \left(u_{s_o}{}^2/c^2\right)}$$

$$g_{22} = -\cot\theta_{s_o}\left[\frac{24 + (2/\gamma)}{\gamma - 1}\,\frac{u_{s_o}{}^2/c^2}{1 - \left(u_{s_o}{}^2/c^2\right)} + \csc^2\theta_{s_o}\right]$$

$$g_{30} = -\frac{2\gamma - 1}{\gamma}\,\frac{\rho^o}{\rho_{s_o}}\,\frac{u_{s_o}}{c}$$

$$g_{31} = 0$$

$$g_{32} = -\frac{\rho^o}{\rho_{s_o}}\,\frac{u_{s_o}}{c}\left[-\frac{4\gamma - 1}{\gamma} - 2\cot^2\theta_{s_o} + 4\cot\theta_{s_o}\csc 2\theta_{s_o} + \right.$$

$$\left.\frac{-8\gamma + 20 - (10/\gamma)}{\gamma - 1}\,\frac{u_{s_o}{}^2/c^2}{1 - \left(u_{s_o}{}^2/c^2\right)}\right]$$

$$h_{10} = \frac{1}{\gamma} \frac{\rho_{s_0}}{\rho^0} \frac{u_{s_0}/c}{1 - \left(u_{s_0}{}^2/c^2\right)}$$

$$h_{11} = \frac{1}{2\gamma} \frac{\rho_{s_0}}{\rho^0} \frac{u_{s_0}/c}{1 - \left(u_{s_0}{}^2/c^2\right)} \cot \theta_{s_0}$$

$$h_{12} = \frac{1}{\gamma} \frac{\rho_{s_0}}{\rho^0} \frac{u_{s_0}/c}{1 - \left(u_{s_0}{}^2/c^2\right)} \left[- \frac{2 + 10\gamma}{\gamma - 1} - \frac{1}{4} \cot^2 \theta_{s_0} + \frac{(8/3) - 6\gamma}{\gamma - 1} \frac{u_{s_0}{}^2/c^2}{1 - \left(u_{s_0}{}^2/c^2\right)} \right]$$

$$h_{20} = -\frac{2}{\gamma} \frac{\rho_{s_0}}{\rho^0} \frac{u_{s_0}/c}{1 - \left(u_{s_0}{}^2/c^2\right)}$$

$$h_{21} = -\frac{8}{\gamma - 1} \frac{\rho_{s_0}}{\rho^0} \frac{u_{s_0}/c}{1 - \left(u_{s_0}{}^2/c^2\right)} \cot \theta_{s_0}$$

$$h_{22} = -\frac{2}{\gamma} \frac{\rho_{s_0}}{\rho^0} \frac{u_{s_0}/c}{1 - \left(u_{s_0}{}^2/c^2\right)} \left[- \frac{6\gamma^2 + 6\gamma + 4}{(\gamma - 1)^2} \frac{u_{s_0}{}^2}{1 - \left(u_{s_0}{}^2/c^2\right)} - \frac{4\gamma}{\gamma - 1} \cot \theta_{s_0} \left(\cot \theta_{s_0} + \csc 2\theta_{s_0} \right) - \frac{3\gamma - 1}{\gamma - 1} \right]$$

$$h_{30} = \frac{\gamma - 1}{2\gamma}$$

$$h_{31} = \frac{\gamma - 1}{4\gamma} \cot \theta_{s_o}$$

$$h_{32} = \frac{\gamma - 1}{2\gamma}\left[-1 - \frac{1}{4}\cot^2\theta_{s_o} + \frac{8\gamma^2 - \frac{28}{3}\gamma - \frac{20}{3}}{(\gamma - 1)^2} \frac{{u_{s_o}}^2/c^2}{1 - \left({u_{s_o}}^2/c^2\right)}\right]$$

APPENDIX B

FURTHER DISCUSSION OF PERTURBATION SCHEME AND SINGULARITY AT NOSE

If a better approximation is desired, it is necessary to look into the nature of the perturbations more closely. Still using the polar coordinates, assume now that each hydrodynamic quantity is representable in a series expansion as follows:

$$u/c = \sum_{n=0}^{\infty} r^n u_n(\theta) \tag{B1}$$

$$v/c = \sum_{n=0}^{\infty} r^n v_n(\theta) \tag{B2}$$

$$\rho = \sum_{n=0}^{\infty} r^n \rho_n(\theta) \tag{B3}$$

while equation (4) serves as the relation among p, ρ, u, and v. The conical solution and the perturbation (13) may be interpreted to be the first two terms of the above series. One then may calculate:

$$\frac{\partial p}{\partial r} = \sum_{n=0}^{\infty} p_{rn} r^n \tag{B4}$$

$$\frac{\partial p}{\partial \theta} = \sum_{n=0}^{\infty} p_{\theta n} r^n \tag{B5}$$

$$\frac{1}{\rho} = \sum_{n=0}^{\infty} \rho_{-n} r^n \tag{B6}$$

where

$$p_{rn} = -\frac{\gamma - 1}{\gamma}\left\{\sum_{k+j=n}\left[\rho_k \sum_{l+m=j}\left(u_l u_{m+1} + v_l v_{m+1}\right) + \frac{k+1}{2}\rho_{k+1}\sum_{l+m=j}\left(u_l u_m + v_l v_m\right)\right] - \frac{n+1}{2}\rho_{n+1}\right\} \tag{B7}$$

$$p_{n\theta} = -\frac{\gamma - 1}{\gamma}\left\{\sum_{k+j=n}\left[\rho_k \sum_{l+m=j}\left(u_l u_m' + v_l v_m'\right) + \frac{\rho_k'}{2}\sum_{l+m=j}\left(u_l u_m + v_l v_m\right)\right] - \frac{\rho_n'}{2}\right\} \tag{B8}$$

$$\left.\begin{aligned} \rho_{-o} &= \frac{1}{\rho_o} \\ \sum_{l+m=n} \rho_{-l}\rho_m &= 0 \qquad (n \neq 0) \end{aligned}\right\} \tag{B9}$$

the primed quantities being the derivatives with respect to θ. It can be shown that the expression for ρ_{-k} in equation (B9) is, in general, of the form

$$\rho_{-k} = \sum_{l+m=k} f_{kl}\left(\rho_o\right)\rho_l \rho_m \tag{B10}$$

with $f_{kl}\left(\rho_o\right)$ a function of ρ_o only. After substitution into the fundamental equations (1) to (3), there follows by equating the coefficients of the kth power of r a set of equations for the $(k + 1)$th-order perturbations:

$$\sum_{l+m=k} \left[u_l u_{m+1}(m + 1) + p_{rm}\rho_{-l}\right] + \sum_{l+m=k+1} v_l\left(u_m' - v_m\right) = 0 \tag{B11}$$

$$\sum_{l+m=k} u_l v_{m+1}(m + 1) + \sum_{l+m=k+1} \left[v_l\left(u_m + v_m'\right) + p_{\theta m}\rho_{-l}\right] = 0 \tag{B12}$$

$$\sum_{l+m=k} \left[\rho_l v_m' + \rho_l' v_m + (n + 2)\rho_l u_m + \rho_l v_m \cot \theta\right] = 0 \tag{B13}$$

In more explicit form, equations (B11) to (B13) may be rewritten as

$$v_o u_{k+1}' + \frac{k\gamma + 1}{\gamma} u_o u_{k+1} - \frac{2\gamma - 1}{\gamma} v_o v_{k+1} + (k + 1) \frac{p_o}{\rho_o^2} \rho_{k+1} =$$

$$Q_{1k}\left(u_o, v_o, \rho_o, \ . \ . \ ., u_k, v_k, \rho_k\right) \tag{B14}$$

$$-\frac{\gamma - 1}{\gamma} u_o u_{k+1}' + \left(\frac{1}{\gamma} v_o - \frac{\gamma - 1}{\gamma} u_o \frac{\rho_o'}{\rho_o}\right) u_{k+1} + \frac{1}{\gamma} v_o v_{k+1}' +$$

$$\left[(k + 2)u_o + \frac{1}{\gamma} v_o' - \frac{\gamma - 1}{\gamma} \frac{\rho_o'}{\rho_o} v_o\right] v_{k+1} + \frac{p_o}{\rho_o^2} \rho_{k+1}' -$$

$$\frac{p_o}{\rho_o^2} \frac{\rho_o'}{\rho_o} \rho_{k+1} = Q_{2k}\left(u_o, v_o, \rho_o, \ . \ . \ ., u_k, v_k, \rho_k\right) \tag{B15}$$

$$(k + 3)u_{k+1} + v_{k+1}' + \left(\frac{\rho_o'}{\rho_o} + \cot\theta\right)v_{k+1} + \frac{v_o}{\rho_o}\rho_{k+1}' + \left[\frac{v_o'}{\rho_o} + \frac{u_o}{\rho_o}(k + 3) + \frac{v_o}{\rho_o}\cot\theta\right]\rho_{k+1} = Q_{3k}\left(u_o, v_o, \rho_o, \ldots, u_k, v_k, \rho_k\right) \tag{B16}$$

where Q_{1k}, Q_{2k}, and Q_{3k} contain the lower-order functions. The Q functions turn out to be zero only for the zeroth- (the conical) and the first-order functions. However, in the general discussion of the kth-order solution, it is only necessary to pick out the terms with strongest singularity.

Again, equations (B14) to (B16) may be put into standard form in parallel with the previous study. Thus,

$$u_{k+1}' = F_{k1}u_{k+1} + F_{k2}v_{k+1} + F_{k3}\rho_{k+1} + \Delta_{F,k} \tag{B17}$$

$$v_{k+1}' = G_{k1}u_{k+1} + G_{k2}v_{k+1} + G_{k3}\rho_{k+1} + \Delta_{G,k} \tag{B18}$$

$$\rho_{k+1}' = H_{k1}u_{k+1} + H_{k2}v_{k+1} + H_{k3}\rho_{k+1} + \Delta_{H,k} \tag{B19}$$

where the Δ's represent combinations of lower-order functions, and the coefficients F, G, and H are given in the following expressions:

$$\left.\begin{aligned}
F_{k1} &= -\frac{k\gamma + 1}{\gamma}\frac{u_o}{v_o} \\
F_{k2} &= \frac{2\gamma - 1}{\gamma} \\
F_{k3} &= -\frac{k + 1}{v_o}\frac{p_o}{\rho_o^{\,2}}
\end{aligned}\right\} \tag{B20}$$

$$
\left.
\begin{aligned}
G_{k1} &= \frac{(k + 3)\dfrac{p_o}{\rho_o} - \left[\dfrac{1}{\gamma} v_o^2 - \dfrac{\gamma - 1}{\gamma} \dfrac{\rho_o'}{\rho_o} u_o v_o + \dfrac{(k\gamma + 1)(\gamma - 1)}{\gamma^2} u_o^2\right]}{\dfrac{1}{\gamma} v_o^2 - \dfrac{p_o}{\rho_o}} \\[2ex]
G_{k2} &= \frac{\dfrac{p_o}{\rho_o}\left(\dfrac{\rho_o'}{\rho_o} + \cot \theta\right) - v_o\left[(k + 2)u_o + \dfrac{1}{\gamma} v_o' - \dfrac{\gamma - 1}{\gamma} \dfrac{\rho_o'}{\rho_o} v_o - \dfrac{(2\gamma - 1)(\gamma - 1)}{\gamma^2} u_o\right]}{\dfrac{1}{\gamma} v_o^2 - \dfrac{p_o}{\rho_o}} \\[2ex]
G_{k3} &= \frac{p_o}{\rho_o^2} \, \frac{v_o' + v_o\left(\dfrac{\rho_o'}{\rho_o} + \cot \theta\right) + \left(2 + \dfrac{k + 1}{\gamma}\right)u_o}{\dfrac{1}{\gamma} v_o^2 - \dfrac{p_o}{\rho_o}}
\end{aligned}
\right\} \qquad \text{(B21)}
$$

$$
\left.
\begin{aligned}
H_{k1} &= \rho_o \, \frac{-\dfrac{(k + 2)}{\gamma} v_o - \dfrac{\gamma - 1}{\gamma} \dfrac{\rho_o'}{\rho_o} u_o + \dfrac{(k\gamma + 1)(\gamma - 1)}{\gamma^2} \dfrac{u_o^2}{v_o}}{\dfrac{1}{\gamma} v_o^2 - \dfrac{p_o}{\rho_o}} \\[2ex]
H_{k2} &= \rho_o \, \frac{(k + 2)u_o + \dfrac{1}{\gamma} v_o' - \left(\dfrac{\rho_o'}{\rho_o} + \dfrac{1}{\gamma} \cot \theta\right)v_o - \dfrac{(2\gamma - 1)(\gamma - 1)}{\gamma^2} u_o}{\dfrac{1}{\gamma} v_o^2 - \dfrac{p_o}{\rho_o}} \\[2ex]
H_{k3} &= \frac{-\dfrac{p_o}{\rho_o}\left[\dfrac{\rho_o'}{\rho_o} - \dfrac{(k + 1)(\gamma - 1)}{\gamma} \dfrac{u_o}{v_o}\right] - \dfrac{1}{\gamma} v_o\left[v_o' + (k + 3)u_o + v_o \cot \theta\right]}{\dfrac{1}{\gamma} v_o^2 - \dfrac{p_o}{\rho_o}}
\end{aligned}
\right\} \qquad \text{(B22)}
$$

The point $\theta = \theta_{s_o}$ is seen to remain a pole for the coefficients F_{k1}, F_{k3}, H_{k1}, and H_{k3} as in the first-order perturbation. Proceeding along a similar line, one writes down the indicial equation for the index α_k as

$$\begin{vmatrix} f_{k1,0} - \alpha_k & 0 & f_{k3,0} \\ 0 & -\alpha_k & 0 \\ h_{k1,0} & 0 & h_{k3,0} - \alpha_k \end{vmatrix} = 0 \tag{B23}$$

where f_{k1}, and so forth are the values of the regularized functions $(\theta - \theta_{s_o})F_{k1}$, and so forth (cf. equation (46)) at $\theta = \theta_{s_o}$. Hence,

$$-\alpha_k\left[\alpha_k^2 - \alpha_k\left(f_{k1,0} + h_{k3,0}\right) + f_{k1,0}h_{k3,0} - f_{k3,0}h_{k1,0}\right] = 0 \tag{B24}$$

It is found by using equation (39) that

$$\left.\begin{aligned} f_{k1,0} &= \frac{k\gamma + 1}{2\gamma} \\ f_{k3,0} &= \frac{k + 1}{2}\,\frac{p_{s_o}}{\rho_{s_o}{}^2 u_{s_o}} \\ h_{k1,0} &= \frac{(k\gamma + 1)(\gamma - 1)}{2\gamma^2}\,\frac{\rho_{s_o}{}^2 u_{s_o}}{p_{s_o}} \\ h_{k3,0} &= \frac{(k + 1)(\gamma - 1)}{2\gamma} \end{aligned}\right\} \tag{B25}$$

The roots of the indicial equation (B24) then are given as

$$\alpha_k = 0, \quad 0, \quad \frac{2k + 1}{2} - \frac{k}{2\gamma}$$

The last root is positive as long as $\gamma > \frac{k}{2k + 1}$, a condition which is satisfied by any real gas. Thus the complementary function of differential equations (B17) to (B19) near $\theta = \theta_{s_o}$ may be put down in series form

$$u_{k+1} = \sum_{n=0}^{\infty} a_{u_k,n}\left(\theta - \theta_{s_o}\right)^n + \sum_{n=0}^{\infty} b_{u_k,n}\left(\theta - \theta_{s_o}\right)^{n+\alpha_k} +$$

$$\log_e \left(\theta - \theta_{s_o}\right) \sum_{n=0}^{\infty} c_{u_k,n}\left(\theta - \theta_{s_o}\right)^n \qquad \text{(B26)}$$

and so on. As in the first-order-perturbation case, the solution for v_{k+1} is again one degree higher in $\left(\theta - \theta_{s_o}\right)$ in the b and c series. In the complementary function, therefore, even for the higher-order functions, nothing worse than a logarithmic term occurs. On the other hand, the particular integrals associated with equations (B17) to (B19) due to the presence of the lower-order functions in the Q functions of equations (B14) to (B16) have stronger singularities at $\theta = \theta_{s_o}$, because each lower-order function contains at least a logarithmic term. To see this, let equations (B17) to (B19) be transformed into the equivalent third-order equation for any of its variables, say u_{k+1} $\left(\text{or } \rho_{k+1}\right)$ as it has been shown to behave worse near the singularity than v_{k+1}. Let $N_k\left(u_o,v_o,\rho_o, \ldots ,u_k,v_k,\rho_k\right)$ be the resultant nonhomogeneous term, arising out of the Q functions through the transformation process. Then if $u_{k+1,1}$, $u_{k+1,2}$, and $u_{k+1,3}$ are the complementary functions represented by the three series in equation (B26), the general solution is obtainable by a variation of constant method. Assuming the solution to be

$$V_{k,1}u_{k+1,1}, \quad V_{k,2}u_{k+1,2}, \quad V_{k,3}u_{k+1,3}$$

one finds

$$\left.\begin{aligned}
V_{k,1} &= \int \frac{W\left(u_{k+1,2}, u_{k+1,3}\right)}{W\left(u_{k+1,1}, u_{k+1,2}, u_{k+1,3}\right)} N \, d\theta \\
V_{k,2} &= \int \frac{W\left(u_{k+1,3}, u_{k+1,1}\right)}{W\left(u_{k+1,1}, u_{k+1,2}, u_{k+1,3}\right)} N \, d\theta \\
V_{k,3} &= \int \frac{W\left(u_{k+1,1}, u_{k+1,2}\right)}{W\left(u_{k+1,1}, u_{k+1,2}, u_{k+1,3}\right)} N \, d\theta
\end{aligned}\right\} \tag{B27}$$

where W stands for the Wronskian. To study the behavior near the singularity, only the dominant terms are of importance. From equation (B26) it is seen that near the singularity

$$u_{k+1,1} \approx \text{Constant}, \quad u_{k+1,2} \approx \left(\theta - \theta_{s_o}\right)^{\alpha_k}, \quad u_{k+1,3} \approx \log\left(\theta - \theta_{s_o}\right)$$

Hence,

$$W\left(u_{k+1,1}, u_{k+1,2}, u_{k+1,3}\right) \approx \begin{vmatrix} 1 & \left(\theta - \theta_{s_o}\right)^{\alpha_k} & \log\left(\theta - \theta_{s_o}\right) \\ 1 & \left(\theta - \theta_{s_o}\right)^{\alpha_k - 1} & \dfrac{1}{\theta - \theta_{s_o}} \\ 1 & \left(\theta - \theta_{s_o}\right)^{\alpha_k - 2} & \dfrac{1}{\left(\theta - \theta_{s_o}\right)^2} \end{vmatrix}$$

$$\approx \left(\theta - \theta_{s_o}\right)^{\alpha_k - 3}$$

$$W\left(u_{k+1,1}, u_{k+1,2}\right) \approx \left(\theta - \theta_{s_o}\right)^{\alpha_k - 1}$$

$$W\left(u_{k+1,2}, u_{k+1,3}\right) \approx \left(\theta - \theta_{s_o}\right)^{\alpha_k - 1} \log\left(\theta - \theta_{s_o}\right)$$

$$W\left(u_{k+1,3}, u_{k+1,1}\right) \approx \left(\theta - \theta_{s_o}\right)^{-1}$$

After substitution of these dominant terms in equation (B27), there follows

$$\left.\begin{aligned}
V_{k,1}u_{k+1,1} &\approx \int\left(\theta - \theta_{s_o}\right)^2 \log_e \left(\theta - \theta_{s_o}\right) N \, d\theta \\
V_{k,2}u_{k+1,2} &\approx \left(\theta - \theta_{s_o}\right)^{\alpha_k} \int \left(\theta - \theta_{s_o}\right)^{2-\alpha_k} N \, d\theta \\
V_{k,3}u_{k+1,3} &\approx \log_e \left(\theta - \theta_{s_o}\right) \int \left(\theta - \theta_{s_o}\right)^2 N \, d\theta
\end{aligned}\right\} \tag{B28}$$

It remains next to find out the singular behaviour of N in the neighborhood of $\theta - \theta_{s_o}$. In the process to reduce to the standard form equations (B17) to (B19), the elimination of v_{k+1}' or ρ_{k+1}' from equations (B15) and (B16) involves a multiplication by v_o. Consequently,

$$\left.\begin{aligned}
\Delta_{F,k} &= Q_{1k}/v_o \\
\Delta_{G,k} &\approx Q_{1k} + v_o Q_{2k} \\
\Delta_{H,k} &\approx Q_{1k} + v_o Q_{3k}
\end{aligned}\right\} \tag{B29}$$

As v_o is known to vary as $\left(\theta - \theta_{s_o}\right)$ near the singularity, unless Q_{2k} or Q_{3k} contains terms of higher-order singularity than $Q_{1k}\Big/\left(\theta - \theta_{s_o}\right)^2$ the main contribution to the singularity of N will be Q_{1k}. A closer examination reveals that the Q functions in equations (B14) to (B16) are all combinations of products of the lower-order functions. With the knowledge that the v function is, in general, one degree higher in $\left(\theta - \theta_{s_o}\right)$

than the corresponding u or ρ function, the leading terms of the Q functions must be included in the following expressions:

For Q_{1k},

$$\sum_{l+m=k} \left[u_l u_m (m + 1) + p_{rm} \rho_{-l} \right]$$

For Q_{2k},

$$\sum_{l+m=k+1} p_{\theta m} \rho_{-l}$$

For Q_{3k},

$$\sum_{l+m=k} \rho_l u_m$$

At once Q_{3k} may be discarded, for it is at most of the same order as Q_{1k}. To compare Q_{1k} and Q_{2k}, formulas (B7) and (B8) for p_{rm} and $p_{\theta m}$ are needed. Meanwhile, let it be assumed that the higher-order functions have stronger or equally strong singularities than their corresponding lower ones. Then the leading terms must be included in

For Q_{1k},

$$\sum_{l+m=k} \rho_{-l} \sum_{p+q=m} \rho_{p+1} \sum_{j=0}^{q} u_j u_{q-j}$$

For Q_{2k},

$$\sum_{l+m=k+1} \rho_{-l} \sum_{p+q=m} \rho_p \sum_{j=0}^{q} u_j' u_{q-j}$$

As the highest terms containing the (k+1)th-order functions are taken out in these expressions, the two summations containing ρ_n become identical and the difference lies in the terms $u_j' u_{q-j}$ and $u_j u_{q-j}$. No more conclusions can be drawn without further knowledge as to the nature of the solutions u_n.

To fix ideas, consider the second-order equations, the Q functions of which contain the zeroth and first-order functions whose forms are known. The Q functions are as follows:

$$
\left.
\begin{aligned}
Q_{12} &= \frac{\gamma - 1}{\gamma}\left[\frac{\rho_1{}^2}{2\rho_o{}^2}\left(1 - u_o{}^2 - v_o{}^2\right) + \frac{\rho_1}{\rho_o}\left(u_1u_o + v_1v_o\right) + \right. \\
&\qquad \left. u_1{}^2 + v_1{}^2\right] - u_1{}^2 - v_1\left(u_1' - v_1\right) \\
Q_{22} &= \frac{\gamma - 1}{\gamma}\left[-\frac{1}{2}\frac{\rho_1{}^2}{\rho_o{}^2}\left(\frac{\rho_o'}{\rho_o} - \frac{\rho_1'}{\rho_1}\right)\left(1 - u_o{}^2 - v_o{}^2\right) - \right. \\
&\qquad \frac{\rho_1}{\rho_o}\left(\frac{\rho_o'}{\rho_o} - \frac{\rho_1'}{\rho_1}\right)\left(u_1u_o + v_1v_o\right) + u_1u_1' + v_1v_1' + \\
&\qquad \left. \frac{1}{2}\frac{\rho_o'}{\rho_o}\left(u_1{}^2 + v_1{}^2\right)\right] - 2u_1v_1 - v_1v_1' \\
Q_{32} &= -\rho_1v_1' - \rho_1'v_1 - 4\rho_1u_1 - \rho_1v_1 \cot\theta
\end{aligned}
\right\} \qquad (B30)
$$

With u_1 and ρ_1 varying as $\log\left(\theta - \theta_{s_o}\right)$ near the singularity,

$$
\left.
\begin{aligned}
Q_{12} &\approx \log^2\left(\theta - \theta_{s_o}\right) \\
Q_{22} &\approx \frac{1}{\left(\theta - \theta_{s_o}\right)} \log\left(\theta - \theta_{s_o}\right) \\
Q_{32} &\approx \log^2\left(\theta - \theta_{s_o}\right)
\end{aligned}
\right\} \qquad (B31)
$$

Thus Q_{12} evidently has a stronger singularity near $\theta = \theta_{s_o}$ than $\left(\theta - \theta_{s_o}\right)^2 Q_{22}$. In this case Δ_{F2} (cf. equation (B29)) contributes most to the function N.

At any rate, proceeding on the discussion of the particular solution, one observes that in twice differentiating equations (B17) to (B19) to arrive at a third-order equation for one dependent variable, the nonhomogeneous term N necessarily will contain second derivatives of the Δ functions. The algebraic operations involving the regularized functions f, g, and h during the process do not affect the nature of the singularity. If the second derivative Δ'' is computed from the leading term in the Δ functions, it is permissible to replace N by Δ'' in equations (B28) for order-of-magnitude study. Again, consider the second-order functions: The leading term of the Δ functions is

$$\frac{Q_{12}}{\theta - \theta_{s_o}} \approx \frac{1}{\theta - \theta_{s_o}} \log^2\left(\theta - \theta_{s_o}\right)$$

Hence,

$$\Delta'' \approx \frac{1}{\left(\theta - \theta_{s_o}\right)^3} \log^2\left(\theta - \theta_{s_o}\right)$$

By substitution into equation (B28),

$$\left.\begin{aligned}
v_{1,1}u_{2,1} &\approx \int \frac{\log^3\left(\theta - \theta_{s_o}\right)}{\theta - \theta_{s_o}}\,d\theta \\
&\approx \log^4\left(\theta - \theta_{s_o}\right) \\
v_{1,2}u_{2,2} &\approx \left(\theta - \theta_{s_o}\right)^{\alpha_k} \int \frac{\log^2\left(\theta - \theta_{s_o}\right)}{\left(\theta - \theta_{s_o}\right)^{\alpha_k+1}}\,d\theta \\
&\approx \log^2\left(\theta - \theta_{s_o}\right) \\
v_{1,3}u_{2,3} &\approx \log\left(\theta - \theta_{s_o}\right) \int \frac{\log^2\left(\theta - \theta_{s_o}\right)}{\theta - \theta_{s_o}}\,d\theta \\
&\approx \log^4\left(\theta - \theta_{s_o}\right)
\end{aligned}\right\} \tag{B32}$$

Thus the second-order function contains terms of the order $\log^4\left(\theta - \theta_{s_o}\right)$, whereas the first-order one contains only $\log\left(\theta - \theta_{s_o}\right)$.

In view of this evidence, one is led to make the assumptions that the leading term in the singularity for the kth-order function takes the form of powers of $\log\left(\theta - \theta_{s_o}\right)$, that is,

$$u_k \approx \left[\log\left(\theta - \theta_{s_o}\right)\right]^{s_k}$$

and that $s_k > s_{k-1}$. Consider the combination $u_j u_{q-j}$ in Q_{1k},

$$u_j u_{q-j} \approx \left[\log\left(\theta - \theta_{s_o}\right)\right]^{s_j + s_{q-j}}$$

while the combination $u_j' u_{q-j}$ in Q_{2k} becomes

$$u_j' u_{q-j} \approx \frac{\left[\log\left(\theta - \theta_{s_o}\right)\right]^{s_j + s_{q-j} - 1}}{\theta - \theta_{s_o}}$$

Therefore Q_{1k} has a singularity much stronger than $\left(\theta - \theta_{s_o}\right)^2 Q_{2k}$, and the contribution to the leading term of N is only by Q_{1k}. To ascertain what choice of the index j will give a maximum value of $s_j + s_{q-j}$ requires, however, more than the monotonic increasing property assumed. For instance, if the second difference of the sequence s_k is assumed to be of the same sign throughout, a positive one requires $j = 0$ and a negative one requires j to be approximately $q/2$ for $s_j + s_{q-j}$ to attain a maximum. If the former is true for the present case, the highest combination of $u_j u_{q-j}$ would be $u_o u_q = \left[\log\left(\theta - \theta_{s_o}\right)\right]^{s_q}$. With a similar argument, the leading term of Q_{1k} is then

$$Q_{1k} \approx \rho_1 u_k$$

$$\approx \left[\log\left(\theta - \theta_{s_o}\right)\right]^{s_k + 1}$$

Hence,

$$\frac{Q_{1k}}{\theta - \theta_{s_0}} \approx \frac{\left[\log\left(\theta - \theta_{s_0}\right)\right]^{s_k+1}}{\theta - \theta_{s_0}}$$

$$\Delta'' \approx \frac{1}{\left(\theta - \theta_{s_0}\right)^3}\left[\log\left(\theta - \theta_{s_0}\right)\right]^{s_k+1}$$

By substitution into equation (B28)

$$\left.\begin{aligned}
V_{k,1}u_{k+1,1} &\approx \int \frac{\left[\log\left(\theta - \theta_{s_0}\right)\right]^{s_k+2}}{\theta - \theta_{s_0}}\,d\theta \\
&\approx \left[\log\left(\theta - \theta_{s_0}\right)\right]^{s_k+3} \\
V_{k,2}u_{k+1,2} &\approx \left(\theta - \theta_{s_0}\right)^{\alpha_k}\int \frac{\left[\log\left(\theta - \theta_{s_0}\right)\right]^{s_k+1}}{\left(\theta - \theta_{s_0}\right)^{\alpha_k+1}}\,d\theta \\
&\approx \left[\log\left(\theta - \theta_{s_0}\right)\right]^{s_k+1} \\
V_{k,3}u_{k+1,3} &\approx \log\left(\theta - \theta_{s_0}\right)\int \frac{\left[\log\left(\theta - \theta_{s_0}\right)\right]^{s_k+1}}{\theta - \theta_{s_0}}\,d\theta \\
&\approx \left[\log\left(\theta - \theta_{s_0}\right)\right]^{s_k+3}
\end{aligned}\right\} \tag{B33}$$

If the solutions in equations (B33) hold true, the second difference of s_k is nil as the sequence s_k is now increasing linearly with k. The combination of $u_j u_{q-j}$ then becomes indifferent to choice of j, so the result (B33) has no contradiction. The derivation is thus justified a posteriori. Formulas (B32) are seen to be given by equations (B33) with $k = 1$, $s_1 = 1$. One concludes by mathematical induction that for the (k+1)th-order function, its singularity has the leading term $0\left\{\left[\log\left(\theta - \theta_{s_0}\right)\right]^{1+3k}\right\}$.

The nature of the differential equations for the kth-order perturbations as assumed by equations (B1) to (B3) having been clarified, it remains next to investigate the proper boundary conditions and the results thereby arrived at. Generalizing equations (23) and (24) one has

$$\left(\sum_{n=1}^{\infty} r^n u_n, \sum_{n=1}^{\infty} r^n v_n, \sum_{n=1}^{\infty} r^n \rho_n\right)_{\theta=\theta_w} = \left(u_w - u_o, v_w - v_o, \rho_w - \rho_o\right)_{\theta=\theta_w} \tag{B34}$$

$$\left(\frac{\sum_{n=0}^{\infty} r^n v_n}{\sum_{n=0}^{\infty} r^n u_n}\right)_{\theta=\theta_s} = \left(r\frac{d\theta_s}{dr}\right)_{\theta=\theta_s} \tag{B35}$$

Since the differential equations must be numerically integrated from the initial point, let equation (B34) be examined first. Expanding into a power series of r near $r = 0$,

$$u_w = u_w\Big|_{\theta=\theta_{w_o}} + \frac{du_w}{d\psi}\Big|_{\theta=\theta_{w_o}}\left(\psi - \theta_{w_o}\right) + \frac{1}{2}\frac{d^2u_w}{d\psi^2}\Big|_{\theta=\theta_{w_o}}\left(\psi - \theta_{w_o}\right)^2 + \ldots +$$

$$\frac{1}{n!}\frac{d^nu_w}{d\psi^n}\Big|_{\theta=\theta_{w_o}}\left(\psi - \theta_{w_o}\right)^n + \ldots \tag{B36}$$

Assume now that the shock-wave shape is regular and also representable in power series in r. Then

$$\psi - \theta_{w_o} = \theta_w - \theta_{w_o} + \tan^{-1}\left(r\,\frac{d\theta_w}{dr}\right)$$

$$= \sum_{n=1}^{\infty} \frac{1}{n!}\,\frac{d^n\theta_{w_o}}{dr^n}\,r^n + \sum_{n=0}^{\infty} \frac{(-1)^n}{2n+1}\left(r\,\frac{d\theta_{w_o}}{dr}\right)^{2n+1}$$

$$= \sum_{n=0}^{\infty} \psi_n r^n$$

where

$$\left.\begin{aligned}
\psi_o &= 0\\
\psi_n &= \frac{(-1)^{\frac{n-1}{2}}}{n}\left(\frac{d\theta_{w_o}}{dr}\right)^n + \frac{1}{n!}\,\frac{d^n\theta_{w_o}}{dr^n} \quad \text{for odd } n\\
\psi_n &= \frac{1}{n!}\,\frac{d^n\theta_{w_o}}{dr^n} \quad \text{for even } n > 0
\end{aligned}\right\} \qquad \text{(B37)}$$

with $d^n\theta_{w_o}/dr^n$ representing the nth derivative of θ_w evaluated at θ_{w_o}. After substitution equation (B36) becomes

$$u_w = \sum_{n=0}^{\infty} \frac{1}{n!}\,\frac{d^n u_{w_o}}{d\psi^n} \sum_{k=0}^{\infty} \sum_{m_1+m_2+\ldots+m_n=k} \psi_{m_1}\psi_{m_2}\ldots\psi_{m_n} r^k$$

with $d^n u_{w_o}/dr^n$ likewise representing the nth derivative of u_w evaluated at θ_{w_o}. Regrouping the terms, one gets the expansion of u_w in ascending powers of r,

$$u_w = \sum_{k=0}^{\infty} \mu_{w,k} r^k \tag{B38}$$

where

$$\mu_{w,k} = \sum_{m_1+m_2+\ldots+m_n=k} \sum_{n=0}^{\infty} \frac{1}{n!} \frac{d^n u_{w_o}}{d\psi^n} \psi_{m_1}\psi_{m_2} \cdots \psi_{m_n} \tag{B39}$$

In a similar way u_o at $\theta = \theta_w$ is obtained:

$$u_o = \sum_{k=0}^{\infty} \mu_{o,k} r^k \tag{B40}$$

where

$$\mu_{o,k} = \sum_{m_1+m_2+\ldots+m_n=k} \sum_{n=0}^{\infty} \frac{1}{n!} \frac{d^n u_{o,o}}{d\theta^n} \theta_{m_1}\theta_{m_2} \cdots \theta_{m_n} \tag{B41}$$

and, in turn,

$$\left.\begin{aligned} \theta_o &= 0 \\ \theta_m &= \frac{1}{m!} \frac{d^n \theta_{w_o}}{dr^m} \end{aligned}\right\} \tag{B42}$$

with $d^n u_{o,o}/d\theta^n$ representing the nth derivative of u_o evaluated at θ_{w_o}. With the help of expansions (B38) and (B41) for u and similar ones for v and ρ, the coefficient of r^n in two sides of equation (B34) may be equated. The proper initial point at the shock wave is then readily derived:

$$\left(u_k, v_k, \rho_k\right)_{\theta=\theta_{w_o}} = \left(\mu_{w,k} - \mu_{o,k}, \nu_{w,k} - \nu_{o,k}, \omega_{w,k} - \omega_{o,k}\right)_{\theta=\theta_{w_o}} \tag{B43}$$

In equation (B43) the ν's and ω's are the coefficients for v and ρ, respectively, corresponding to the μ's for u defined by equations (B39) and (B41). They are defined in exactly the same manner except that u is to be replaced by the variable in question.

With the initial point specified by equation (B43), the differential equations may be integrated numerically and the three arbitrary constants in the series form near the singular point $\theta = \theta_{s_o}$ determined. Consider now the quantities when the body is reached. The left-hand side of equation (B35) may be rewritten as

$$\frac{\sum_{n=0}^{\infty} r^n v_n}{\sum_{n=0}^{\infty} r^n u_n} = \sum_{n=0}^{\infty} \chi_n(\theta) r^n \tag{B44}$$

by defining

$$\chi_n = \sum_{l+m=n} v_l u_{-m} \tag{B45}$$

and

$$\left. \begin{aligned} u_{-o} &= \frac{1}{u_o} \\ \sum_{l+m=n} u_m u_{-l} &= 0 \quad \text{for} \quad n \neq 0 \end{aligned} \right\} \tag{B46}$$

Each term in χ_n therefore contains, in general, quantities of the form $\left(\theta - \theta_{s_o}\right)\left[\log_e \left(\theta - \theta_{s_o}\right)\right]^k$ near $\theta - \theta_{s_o}$. The important conclusion now presents itself: If the shock-wave shape is assumed to be regular, the body shape must have a singular point at the vertex. An expansion of the body shape in power series of r in that neighborhood is not possible. The previous method cannot be used to obtain the higher-order derivatives of $\theta_s = \theta_s(r)$.

The approximate behavior of the singular body shape near the vertex to produce the assumed regular shock may be seen by the following consideration. Writing out the first three terms of equation (B44), one has

$$\left(r\,\frac{d\theta_s}{dr}\right)_{\theta=\theta_s} = \frac{v_o}{u_o} + r_s\left(\frac{v_1}{u_o} - \frac{v_o u_1}{u_o^2}\right) + r_s^2\left[\frac{v_2}{u_o} - \frac{v_1 u_1}{u_o^2} - \frac{v_o}{u_o}\left(-\frac{u_1^2}{u_o} + \frac{u_2}{u_o}\right)\right] + \ldots \tag{B47}$$

As previously shown,

$$v_o \approx O\left(\theta - \theta_{s_o}\right)$$

$$u_o \approx O(1)$$

$$v_1 \approx O\left[\left(\theta - \theta_{s_o}\right)\log_e\left(\theta - \theta_{s_o}\right)\right] + O(1)$$

$$u_1 \approx O\left[\log_e\left(\theta - \theta_{s_o}\right)\right]$$

$$v_2 \approx O\left[\left(\theta - \theta_{s_o}\right)\log_e^4\left(\theta - \theta_{s_o}\right)\right] + O(1)$$

$$u_2 \approx O\left[\log_e^4\left(\theta - \theta_{s_o}\right)\right]$$

Equation (B47) thus becomes

$$\left(r\,\frac{d\theta_s}{dr}\right)_{\theta=\theta_s} \approx O\left(\theta - \theta_{s_o}\right) + r_s\left\{O(1) + O\left[\left(\theta - \theta_{s_o}\right)\log_e\left(\theta - \theta_{s_o}\right)\right]\right\} + r_s^2\left\{O(1) + O\left[\left(\theta - \theta_{s_o}\right)\log_e\left(\theta - \theta_{s_o}\right)\right] + O\left[\left(\theta - \theta_{s_o}\right)\log_e^2\left(\theta - \theta_{s_o}\right)\right] + O\left[\left(\theta - \theta_{s_o}\right)\log_e^4\left(\theta - \theta_{s_o}\right)\right]\right\} + \ldots$$

or

$$\left(r \frac{d\theta_s}{dr}\right)_{\theta=\theta_s} \approx O\left(\theta - \theta_{s_o}\right) + r_s\left\{O\left[\left(\theta - \theta_{s_o}\right) \log_e \left(\theta - \theta_{s_o}\right)\right] + O(1)\right\} +$$

$$r_s^2\left\{O\left[\left(\theta - \theta_{s_o}\right) \log_e^4\left(\theta - \theta_{s_o}\right)\right] + O(1)\right\} + \ldots \quad \text{(B48)}$$

The derivative $\left.\frac{d\theta_s}{dr}\right|_{\theta=\theta_{s_o}}$ having been shown to be a finite quantity at the surface, one may assume

$$\theta - \theta_{s_o} \approx \left.\frac{d\theta_s}{dr}\right|_{\theta=\theta_{s_o}} r_s + \text{Higher-order terms} \quad \text{(B49)}$$

Hence,

$$r \left.\frac{d\theta_s}{dr}\right|_{\theta=\theta_s} \approx r_s\left[O(1) + O\left(r_s \log_e r_s\right)\right] +$$

$$r_s^2\left[O\left(r_s \log_e^4 r_s\right) + \ldots\right] + \ldots$$

$$r \left.\frac{d\theta_s}{dr}\right|_{\theta=\theta_s} \approx r_s O(1) + O\left(r_s^2 \log_e r_s\right) + O\left(r_s^3 \log_e^4 r_s\right) + \ldots \quad \text{(B50)}$$

Equation (B50) indicates that the body surface as defined by $\theta_s = \theta_s(r)$ must have logarithmic singularities near the vertex. Expression (B49) is also found to lead to no inconsistency.

Conversely, in the usual case when a regular body is given, the shock wave cannot be represented regularly without contradiction. It must have a singularity at the vertex. The nature of the singularity presumably would likewise be logarithmic.

The existence of a finite first derivative $d\theta_s/dr$ as obtained in the section "Integration of Perturbation Equations" is fortunate. By regarding the perturbation as the asymptotic solution which is correct when $r \longrightarrow 0$, the ratio of the initial radii of curvature may be found in spite of the singularities when higher order is considered. It may be pointed out that the "smallness" of r should be measured by a proper scale, which, in this case, is obviously either R_{w_o} or R_{s_o} neither of which, by hypothesis, should be zero. In fact, one may recall that in the above-mentioned section when the first-order perturbation equations are reduced to dimensionless form, the expansion becomes:

$$\frac{u}{c} = \frac{u_o}{c} + \frac{1}{2}\frac{r}{R_{w_o}}\xi$$

and similar expressions can be obtained for v and ρ. The appearance of the parameter r/R_{w_o} verifies the choice of scale stated above.

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TABLE 1.- COEFFICIENT FUNCTIONS F, G, AND H AND

VARIABLES ξ, η, AND ζ

θ (deg)	F_1	F_3	G_1	G_2	G_3	H_1	H_2	H_3	ξ	η	ζ
$\theta_{s_0} = 10^\circ$; $u_{s_0}/c = 0.40$; $M^\circ = 1.0901$											
68.653	0.2853	0.2861	-68.2270	-3.82165	-3.7282	-167.25	-9.6312	-9.484	-0.021	0.659	1.660
68	.2957	.2878	-44.0644	-7.7142	-2.4677	-104.452	-19.770	-6.219	-.037	.751	1.896
67	.3115	.2905	-29.7930	-7.6336	-1.7233	-70.4940	-19.438	-4.275	-.066	.906	2.293
66	.3274	.2932	-22.9179	-7.0077	-1.3655	-53.1359	-17.906	-3.333	-.100	1.061	2.689
65	.3435	.2960	-18.7736	-6.4205	-1.1503	-42.6458	-16.454	-2.759	-.140	1.214	3.083
64	.3598	.2988	-15.9686	-5.9278	-1.0046	-35.5294	-15.228	-2.365	-.185	1.365	3.475
63	.3764	.3019	-13.9308	-5.5186	-.8984	-30.3492	-14.209	-2.074	-.235	1.515	3.866
62	.3933	.3051	-12.3772	-5.1754	-.8173	-26.3885	-13.352	-1.848	-.291	1.664	4.257
61	.4105	.3085	-11.1506	-4.8840	-.7528	-23.2526	-12.623	-1.666	-.352	1.812	4.648
60	.4281	.3119	-10.1561	-4.6336	-.7004	-20.6999	-11.993	-1.514	-.418	1.958	5.040
59	.4461	.3157	-9.3328	-4.4158	-.6565	-18.5809	-11.445	-1.385	-.489	2.103	5.432
58	.4645	.3196	-8.6397	-4.2244	-.6193	-16.7912	-10.962	-1.274	-.565	2.247	5.825
57	.4833	.3237	-8.0481	-4.0550	-.5873	-15.2550	-10.531	-1.175	-.646	2.390	6.219
56	.5026	.3280	-7.5374	-3.9040	-.5595	-13.9248	-10.146	-1.087	-.732	2.531	6.614
55	.5225	.3324	-7.0922	-3.7686	-.5349	-12.7579	-9.7979	-1.007	-.823	2.671	7.010
54	.5428	.3372	-6.7007	-3.6463	-.5132	-11.7262	-9.4802	-.935	-.919	2.809	7.407
53	.5638	.3422	-6.3542	-3.5357	-.4937	-10.8071	-9.1904	-.869	-1.019	2.946	7.805
52	.5854	.3474	-6.0453	-3.4354	-.4762	-9.9824	-8.9258	-.807	-1.124	3.081	8.204
51	.6077	.3530	-5.7685	-3.3440	-.4603	-9.2371	-8.6801	-.749	-1.234	3.214	8.604
50	.6308	.3589	-5.5192	-3.2606	-.4458	-8.5611	-8.4535	-.695	-1.348	3.346	9.005
49	.6546	.3650	-5.2937	-3.1843	-.4325	-7.9435	-8.2425	-.643	-1.467	3.476	9.407
48	.6793	.3716	-5.0889	-3.1147	-.4204	-7.3762	-8.0448	-.594	-1.591	3.604	9.810
47	.7049	.3785	-4.9023	-3.0511	-.4092	-6.8452	-7.8607	-.547	-1.719	3.729	10.213
46	.7315	.3858	-4.7316	-2.9929	-.3988	-6.3711	-7.6881	-.502	-1.851	3.852	10.617
45	.7592	.3935	-4.5751	-2.9399	-.3892	-5.9212	-7.5249	-.458	-1.988	3.973	11.021
44	.7881	.4017	-4.4313	-2.8917	-.3803	-5.5022	-7.3714	-.416	-2.129	4.092	11.426
43	.8182	.4104	-4.2987	-2.8482	-.3719	-5.1094	-7.2254	-.374	-2.274	4.209	11.831
42	.8497	.4196	-4.1763	-2.8089	-.3641	-4.7402	-7.0875	-.334	-2.423	4.323	12.236
41	.8828	.4295	-4.0630	-2.7739	-.3569	-4.3914	-6.9558	-.294	-2.576	4.435	12.642
40	.9175	.4399	-3.9581	-2.7429	-.3500	-4.0611	-6.8299	-.255	-2.733	4.545	13.048
39	.9540	.4511	-3.8606	-2.7159	-.3436	-3.7471	-6.7094	-.216	-2.893	4.652	13.454
38	.9925	.4632	-3.7700	-2.6928	-.3375	-3.4473	-6.5935	-.177	-3.057	4.757	13.860
37	1.0333	.4760	-3.6857	-2.6736	-.3318	-3.1599	-6.4819	-.137	-3.225	4.860	14.266
36	1.0765	.4898	-3.6071	-2.6575	-.3265	-2.8838	-6.3745	-.098	-3.396	4.961	14.672
35	1.1224	.5046	-3.5337	-2.6468	-.3214	-2.6167	-6.2695	-.059	-3.571	5.060	15.078
34	1.1714	.5206	-3.4653	-2.6395	-.3166	-2.3577	-6.1675	-.019	-3.749	5.157	15.484
33	1.2238	.5380	-3.4014	-2.6363	-.3120	-2.1052	-6.0678	.022	-3.931	5.253	15.890
32	1.2801	.5569	-3.3417	-2.6374	-.3078	-1.8581	-5.9700	.064	-4.116	5.347	16.296
31	1.3408	.5774	-3.2859	-2.6430	-.3037	-1.6145	-5.8729	.107	-4.304	5.440	16.701
30	1.4065	.5998	-3.2337	-2.6532	-.2998	-1.3734	-5.7767	.152	-4.495	5.532	17.106
29	1.4780	.6244	-3.1849	-2.6685	-.2962	-1.1332	-5.6805	.199	-4.690	5.624	17.510
28	1.5561	.6518	-3.1393	-2.6894	-.2928	-.8923	-5.5830	.248	-4.888	5.716	17.914
27	1.6420	.6820	-3.0966	-2.7157	-.2895	-.6488	-5.4842	.300	-5.089	5.808	18.317
26	1.7370	.7157	-3.0568	-2.7487	-.2864	-.4011	-5.3833	.355	-5.293	5.901	18.719
25	1.8430	.7535	-3.0196	-2.7887	-.2834	-.1460	-5.2790	.415	-5.501	5.996	19.120
24	1.9621	.7964	-2.9849	-2.8364	-.2806	.1187	-5.1704	.479	-5.712	6.094	19.519
23	2.0972	.8455	-2.9527	-2.8927	-.2780	.3968	-5.0554	.549	-5.927	6.196	19.917
22	2.2521	.9022	-2.9227	-2.9588	-.2754	.6928	-4.9328	.628	-6.145	6.303	20.313
21	2.4322	.9684	-2.8948	-3.0360	-.2730	1.0136	-4.8007	.717	-6.367	6.416	20.706
20	2.6446	1.0472	-2.8693	-3.1257	-.2708	1.3655	-4.6555	.820	-6.593	6.537	21.095
19	2.8998	1.1422	-2.8457	-3.2304	-.2686	1.7619	-4.4951	.940	-6.824	6.669	21.479
18	3.2135	1.2598	-2.8240	-3.3523	-.2666	2.2196	-4.3146	1.083	-7.059	6.814	21.856
17	3.6101	1.4091	-2.8044	-3.4947	-.2648	2.7652	-4.1085	1.261	-7.300	6.975	22.225
16	4.1304	1.6060	-2.7867	-3.6617	-.2631	3.4423	-3.8690	1.491	-7.547	7.157	22.582
15	4.8474	1.8781	-2.7710	-3.8588	-.2616	4.3299	-3.5870	1.801	-7.800	7.365	22.922
14	5.9071	2.2819	-2.7574	-4.0955	-.2602	5.5845	-3.2489	2.254	-8.061	7.607	23.239
13	7.6499	2.9481	-2.7461	-4.3741	-.2590	7.5661	-2.8319	2.983	-8.331	7.891	23.524
12	11.0961	4.2683	-2.7371	-4.7159	-.2580	11.3486	-2.3105	4.408	-8.612	8.231	23.764
11	21.3433	8.1999	-2.7312	-5.1383	-.2574	22.3217	-1.6397	8.597	-8.907	8.645	23.934
10	∞	∞	-2.7289	-5.6713	-.2571	∞	--------	∞	------	9.156	------

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TABLE 1.- COEFFICIENT FUNCTIONS F, G, AND H AND

VARIABLES ξ, η, AND ζ - Continued

θ (deg)	F_1	F_3	G_1	G_2	G_3	H_1	H_2	H_3	ξ	η	ζ
$\theta_{s_0} = 10^\circ$; $u_{s_0}/c = 0.55$; $M^o = 1.595$											
39.708	0.8848	0.2585	-55.820	-9.400	-8.246	-151.086	-26.308	-23.345	-0.023	1.840	5.142
39.5	.8944	.2602	-37.099	-17.078	-5.759	-98.615	-48.100	-16.165	-.037	2.068	5.785
39	.9161	.2637	-21.831	-16.674	-3.539	-55.704	-47.282	-9.731	-.078	2.676	7.498
38.5	.9372	.2673	-15.989	-14.435	-2.631	-39.218	-41.128	-7.071	-.130	3.282	9.202
38	.9584	.2710	-12.825	-12.626	-2.123	-30.244	-36.118	-5.576	-.192	3.889	10.905
37.5	.9798	.2748	-10.817	-11.231	-1.794	-24.305	-32.393	-4.599	-.265	4.500	12.618
37	1.0016	.2786	-9.421	-10.139	-1.562	-20.521	-29.184	-3.905	-.349	5.116	14.345
36.5	1.0238	.2826	-8.389	-9.264	-1.389	-17.546	-26.719	-3.382	-.444	5.739	16.081
36	1.0466	.2867	-7.594	-8.547	-1.255	-15.236	-24.710	-2.930	-.550	6.369	17.831
35.5	1.0700	.2910	-6.961	-7.950	-1.147	-13.383	-23.022	-2.638	-.667	7.006	19.598
35	1.0940	.2954	-6.444	-7.444	-1.059	-11.858	-21.585	-2.362	-.795	7.652	21.388
34	1.1441	.3048	-5.6511	-6.6369	-.9223	-9.4803	-19.2722	-1.9256	-1.085	8.970	25.027
33	1.1975	.3149	-5.0695	-6.0210	-.8215	-7.6950	-17.4840	-1.5931	-1.422	10.323	28.745
32	1.2547	.3259	-4.6246	-5.5377	-.7439	-6.2890	-16.0553	-1.3273	-1.806	11.713	32.542
31	1.3161	.3378	-4.2734	-5.1503	-.6824	-5.1392	-14.8838	-1.1067	-2.240	13.141	36.418
30	1.3825	.3508	-3.9891	-4.8351	-.6323	-4.1691	-13.9020	-.9180	-2.724	14.610	40.373
29	1.4545	.3652	-3.7546	-4.5761	-.5908	-3.3285	-13.0636	-.7522	-3.260	16.121	44.405
28	1.5332	.3810	-3.5581	-4.3622	-.5559	-2.5818	-12.3361	-.6028	-3.850	17.677	48.511
27	1.6196	.3985	-3.3914	-4.1855	-.5262	-1.9037	-11.6948	-.4652	-4.495	19.279	52.689
26	1.7152	.4181	-3.2485	-4.0403	-.5006	-1.2745	-11.1220	-.3360	-5.196	20.932	56.936
25	1.8216	.4401	-3.1250	-3.9226	-.4783	-.6780	-10.6027	-.2117	-5.956	22.640	61.250
24	1.9413	.4651	-3.0174	-3.8295	-.4588	-.0999	-10.1252	-.0899	-6.777	24.408	65.625
23	2.0768	.4937	-2.9232	-3.7589	-.4417	.4724	-9.6795	.0320	-7.661	26.243	70.056
22	2.2323	.5267	-2.8403	-3.7099	-.4265	1.0522	-9.2566	.1569	-8.610	28.154	74.536
21	2.4129	.5652	-2.7671	-3.6818	-.4130	1.6541	-8.8484	.2879	-9.627	30.150	79.057
20	2.6259	.6110	-2.7023	-3.6749	-.4010	2.2958	-8.4459	.4288	-10.716	32.244	83.605
19	2.8818	.6664	-2.6449	-3.6901	-.3903	3.0004	-8.0414	.5846	-11.880	34.453	88.158
18	3.1963	.7348	-2.5941	-3.7291	-.3808	3.7994	-7.6251	.7625	-13.123	36.795	92.703
17	3.5937	.8218	-2.5492	-3.7945	-.3724	4.7404	-7.1857	.9733	-14.450	39.298	97.233
16	4.115	.936	-2.5097	-3.8902	-.3649	5.901	-6.709	1.235	-15.87	42.00	101.65
15	4.833	1.095	-2.4753	-4.0216	-.3584	7.417	-6.177	1.577	-17.39	44.94	105.92
14	5.895	1.330	-2.4456	-4.1968	-.3528	9.559	-5.565	2.062	-19.01	48.17	109.97
13	7.639	1.719	-2.4209	-4.4271	-.3482	12.947	-4.838	-2.829	-20.75	51.76	113.64
12	11.088	2.488	-2.4013	-4.7295	-.3446	19.435	-3.944	4.295	-22.62	55.83	116.76
11	21.338	4.781	-2.3879	-5.1301	-.3422	38.260	-2.804	8.532	-24.65	60.54	119.01
10	∞	∞	-------	-------	-------	∞	-------	∞	------	66.10	------
$\theta_{s_0} = 10^\circ$; $u_{s_0}/c = 0.70$; $M^o = 2.387$											
26.363	1.5806	0.21407	-20.257	-5.6300	-3.9297	-64.316	-19.717	-14.104	-0.059	2.617	9.083
26	1.6249	.21827	-12.340	-10.045	-2.7304	-36.016	-35.746	-9.5207	-.094	2.952	10.275
25.5	1.6847	.22412	-8.0921	-10.219	-1.9900	-25.118	-36.753	-6.6516	-.150	3.452	12.034
25.0	1.7453	.23022	-6.1913	-9.4852	-1.5947	-13.359	-34.364	-5.0922	-.215	3.960	13.810
24.5	1.8079	.23665	-5.0940	-8.7142	-1.3448	-9.1126	-31.741	-4.0860	-.288	4.469	15.584
24.0	1.8734	.24346	-4.3795	-8.0350	-1.1709	-6.2396	-29.377	-3.3698	-.370	4.980	17.352
23.5	1.9423	.25071	-3.8773	-7.4573	-1.0424	-4.1255	-27.325	-2.8263	-.461	5.494	19.118
23.0	2.0152	.25845	-3.5051	-6.9679	-.94313	-2.4707	-25.553	-2.3940	-.561	6.012	20.883
22.5	2.0928	.26677	-3.2182	-6.5520	-.86416	-1.1109	-24.010	-2.0377	-.670	6.535	22.648
22.0	2.1757	.27571	-2.9904	-6.1966	-.79969	.0512	-22.658	-1.7352	-.789	7.063	24.414
21.5	2.2646	.28538	-2.8050	-5.8913	-.74606	1.0781	-21.460	-1.4719	-.917	7.598	26.181
21.0	2.3605	.29587	-2.6514	-5.6282	-.70077	2.0126	-20.389	-1.2378	-1.054	8.140	27.949
20.5	2.4642	.30728	-2.5220	-5.4006	-.66200	2.8863	-19.421	-1.0255	-1.201	8.691	29.718
20.0	2.5772	.31977	-2.4116	-5.2037	-.62849	3.7229	-18.538	-.82923	-1.357	9.251	31.487
19.5	2.7007	.33350	-2.3162	-5.0334	-.59925	4.4657	-17.725	-.64472	-1.523	9.820	33.255
19.0	2.8365	.34866	-2.2331	-4.8868	-.57355	5.3585	-16.970	-.46826	-1.699	10.400	35.019
18.5	2.9868	.36522	-2.1601	-4.7614	-.55082	6.1902	-16.260	-.29664	-1.886	10.992	36.779
18.0	3.1542	.38438	-2.0953	-4.6553	-.53062	7.0527	-15.587	-.12688	-2.083	11.597	38.532
17.5	3.3422	.40562	-2.0376	-4.5673	-.51259	7.9636	-14.943	.04395	-2.291	12.216	40.275
17.0	3.5550	.42978	-1.9859	-4.4963	-.49645	8.9410	-14.320	.21892	-2.510	12.851	42.005
16.5	3.7984	.45748	-1.9393	-4.4416	-.48195	10.009	-13.710	.40143	-2.740	13.303	43.717
16.0	4.0798	.48964	-1.8972	-4.4031	-.46893	11.198	-13.106	.59562	-2.981	14.173	45.406
15.5	4.4095	.52742	-1.8589	-4.3807	-.45722	12.544	-12.501	.80648	-3.234	14.865	47.067
15	4.8018	.57252	-1.8241	-4.3746	-.44670	14.102	-11.887	1.0408	-3.500	15.581	48.692
14.5	5.2774	.62732	-1.7924	-4.3857	-.43727	15.945	-11.256	1.3077	-3.778	16.324	50.273
14	5.8673	.69550	-1.7634	-4.4150	-.42886	18.184	-10.599	1.6208	-4.069	17.097	51.798
13.5	6.6202	.78271	-1.7371	-4.4640	-.42139	20.988	-9.9056	2.0007	-4.374	17.905	53.256
13	7.6170	.89844	-1.7133	-4.5350	-.41485	24.645	-9.1623	2.4814	-4.694	18.753	54.630
12.5	9.0035	1.0597	-1.6919	-4.6308	-.40920	29.661	-8.3540	3.1232	-5.029	19.647	55.900
12	11.071	1.3007	-1.6730	-4.7550	-.40445	37.049	-7.4628	4.0455	-5.380	20.594	57.041
11.5	14.500	1.7010	-1.6571	-4.9130	-.40061	49.168	-6.4646	5.5262	-5.748	21.605	58.021
11	21.329	2.4991	-1.6444	-5.1115	-.39774	73.083	-5.3283	8.3932	-6.134	22.691	58.799
10.5	41.755	4.8887	-1.6356	-5.3599	-.39591	144.09	-4.0128	16.780	-6.540	23.868	59.322
10	∞	∞	-1.6323	-5.6713	-.39526	--------	-------	-------	-----	25.155	-----

TABLE 1.- COEFFICIENT FUNCTIONS F, G, AND H AND VARIABLES ξ, η, AND ζ - Continued

θ (deg)	F_1	F_3	G_1	G_2	G_3	H_1	H_2	H_3	ξ	η	ζ
$\theta_{s_o} = 10^o$; $u_{s_o}/c = 0.80$; $M^o = 3.30$											
20.184	2.3954	0.1711	-9.3750	-3.8602	-2.0188	-34.084	-18.184	-9.690	-0.110	2.965	13.457
20.0	2.4458	.1738	-7.1249	-5.5008	-1.7511	-22.652	-26.212	-8.199	-.129	3.091	14.057
19.75	2.5149	.1776	-5.2801	-6.4768	-1.4976	-12.965	-31.141	-6.767	-.156	3.273	14.917
19.5	2.5854	.1816	-4.1433	-6.8226	-1.3175	-6.726	-33.019	-5.732	-.185	3.461	15.801
19.25	2.6580	.1857	-3.3839	-6.8904	-1.1822	-2.335	-33.514	-4.939	-.216	3.653	16.691
19.0	2.7331	.1900	-2.8469	-6.8278	-1.0764	.963	-33.330	-4.305	-.248	3.847	17.583
18.75	2.8111	.1945	-2.4506	-6.7030	-.9911	3.568	-32.806	-3.781	.282	4.042	18.473
18.5	2.8925	.1993	-2.1482	-6.5500	-.9207	5.715	-32.109	-3.338	-.318	4.237	19.357
18.25	2.9781	.2044	-1.9113	-6.3867	-.8615	7.555	-31.328	-2.954	-.355	4.433	20.236
18.0	3.0679	.2097	-1.7216	-6.2228	-.8111	9.175	-30.509	-2.616	-.394	4.630	21.111
17.5	3.2625	.2214	-1.4385	-5.9112	-.7296	12.003	-28.866	-2.040	-.478	5.027	22.843
17.0	3.4813	.2346	-1.2390	-5.6337	-.6665	14.543	-27.284	-1.554	-.568	5.428	24.553
16.5	3.7300	.2497	-1.0914	-5.3943	-.6162	16.993	-25.788	-1.124	-.665	5.834	26.237
16.0	4.0163	.2673	-.9780	-5.1922	-.5755	19.497	-24.369	-.728	-.770	6.247	27.891
15.5	4.3507	.2879	-.8879	-5.0254	-.5418	22.181	-23.014	-.349	-.882	6.668	29.510
15.0	4.7476	.3125	-.8143	-4.8920	-.5137	25.178	-21.703	.029	-1.001	7.098	31.085
14.5	5.2276	.3424	-.7526	-4.7908	-.4901	28.644	-20.415	.421	-1.128	7.539	32.612
14.0	5.8219	.3797	-.6997	-4.7211	-.4701	32.796	-19.128	.845	-1.263	7.993	34.082
13.5	6.5792	.4272	-.6536	-4.6834	-.4531	37.977	-17.817	1.327	-1.406	8.463	35.487
13.0	7.5806	.4904	-.6128	-4.6791	-.4388	44.694	-16.451	1.902	-1.557	8.951	36.813
12.5	8.972	.578	-.5764	-4.7110	-.4270	53.902	-14.996	2.634	-1.716	9.461	38.039
12.0	11.045	.710	-.5440	-4.7836	-.4174	67.454	-13.417	3.645	-1.884	9.997	39.143
11.5	14.479	.929	-.5158	-4.9035	-.4097	89.692	-11.660	5.215	-2.062	10.565	40.094
11.0	21.314	1.364	-.4926	-5.0805	-.4040	133.568	-9.658	8.176	-2.250	11.173	40.849
10.5	41.748	2.699	-.4760	-5.3293	-.4005	263.760	-7.318	16.663	-2.449	11.830	41.359
10	∞	∞	-------	-------	-------	∞	-------	∞	------	12.548	------
$\theta_{s_o} = 10^o$; $u_{s_o}/c = 0.90$; $M^o = 5.42$											
15.0128	4.499	0.109	-1.111	-2.808	-0.824	22.64	-26.13	-6.66	-0.197	3.045	23.321
15.0	4.511	.108	-1.020	-2.881	-.818	23.81	-26.81	-6.57	-.198	3.051	23.375
14.8	4.708	.113	.109	-3.742	-.736	39.41	-34.72	-5.50	-.218	3.151	24.234
14.6	4.918	.118	.850	-4.239	-.674	51.43	-39.31	-4.62	-.239	3.256	25.121
14.4	5.144	.122	1.357	-4.527	-.622	61.36	-41.97	-3.88	-.260	3.365	26.018
14.2	5.388	.127	1.720	-4.693	-.580	70.15	-43.38	-3.24	-.282	3.477	26.919
14.0	5.653	.133	1.985	-4.783	-.545	78.28	-44.01	-2.66	-.305	3.591	27.816
13.8	5.943	.139	2.185	-4.826	-.516	86.13	-44.12	-2.13	-.328	3.707	28.704
13.6	6.263	.146	2.337	-4.840	-.491	93.98	-43.85	-1.64	-.352	3.824	29.580
13.4	6.617	.154	2.454	-4.837	-.470	102.07	-43.32	-1.17	-.377	3.943	30.442
13.2	7.013	.162	2.547	-4.825	-.451	110.62	-42.58	-.71	-.403	4.063	31.288
13.0	7.458	.172	2.625	-4.808	-.434	119.88	-41.68	-.25	-.430	4.185	32.115
12.8	7.964	.183	2.690	-4.790	-.420	130.09	-40.65	.20	-.457	4.309	32.921
12.6	8.545	.195	2.744	-4.775	-.407	141.53	-39.52	.68	-.485	4.434	33.704
12.4	9.219	.210	2.789	-4.765	-.395	154.56	-38.28	1.18	-.514	4.561	34.461
12.2	10.014	.228	2.828	-4.761	-.385	169.66	-36.94	1.72	-.544	4.690	35.190
12.0	10.962	.249	2.864	-4.766	-.377	187.51	-35.50	2.32	-.574	4.822	35.890
11.8	12.116	.275	2.897	-4.781	-.369	209.08	-33.96	2.99	-.605	4.956	36.555
11.6	13.556	.307	2.927	-4.807	-.362	235.82	-32.31	3.77	-.637	5.093	37.182
11.4	15.396	.348	2.954	-4.846	-.355	269.85	-30.54	4.72	-.670	5.233	37.768
11.2	17.846	.403	2.979	-4.899	-.350	314.81	-28.63	5.91	-.704	5.377	38.311
11.0	21.268	.479	3.002	-4.969	-.346	377.47	-26.56	7.50	-.739	5.525	38.807
10.8	26.388	.594	3.022	-5.058	-.343	470.76	-24.31	9.78	-.774	5.678	39.250
10.6	34.904	.785	3.040	-5.169	-.340	625.42	-21.86	13.44	-.810	5.836	39.636
10.4	51.950	1.167	3.055	-5.305	-.338	934.09	-19.16	20.56	-.846	6.000	39.961
10.2	102.985	2.312	3.065	-5.471	-.337	1856.35	-16.18	41.47	-.882	6.171	40.228
10.0	∞	∞	-------	-------	-------	∞	-------	∞	∞	6.349	∞

NACA

TABLE 1.- COEFFICIENT FUNCTIONS F, G, AND H AND

VARIABLES ξ, η, AND ζ - Continued

θ (deg)	F_1	F_3	G_1	G_2	G_3	H_1	H_2	H_3	ξ	η	ζ
					$\theta_{s_0} = 20^\circ$; $u_{s_0}/c = 0.35$; $M^0 = 1.216$						
68.507	0.3433	0.2694	-10.3845	-0.5802	-0.4806	-24.782	-1.725	-1.458	-0.164	0.673	1.932
68	.3524	.2711	-9.8154	-.7826	-.4593	-23.120	-2.341	-1.375	-.176	.670	1.932
67	.3704	.2746	-8.8945	-1.0679	-.4250	-20.418	-3.218	-1.237	-.199	.667	1.938
66	.3889	.2782	-8.1649	-1.2567	-.3979	-18.264	-3.806	-1.126	-.222	.663	1.948
65	.4077	.2819	-7.5696	-1.3858	-.3759	-16.493	-4.214	-1.032	-.245	.660	1.961
64	.4269	.2858	-7.0725	-1.4763	-.3575	-15.005	-4.504	-.951	-.268	.656	1.975
63	.4466	.2899	-6.6502	-1.5405	-.3419	-13.729	-4.712	-.880	-.291	.652	1.989
62	.4669	.2943	-6.2861	-1.5864	-.3283	-12.620	-4.862	-.816	-.313	.647	2.003
61	.4877	.2988	-5.9686	-1.6191	-.3165	-11.644	-4.970	-.758	-.335	.642	2.017
60	.5090	.3036	-5.6889	-1.6422	-.3060	-10.777	-5.046	-.705	-.357	.636	2.031
59	.5311	.3087	-5.4406	-1.6582	-.2966	-9.997	-5.098	-.656	-.379	.630	2.044
58	.5539	.3141	-5.2184	-1.6687	-.2881	-9.292	-5.131	-.609	-.401	.623	2.056
57	.5775	.3198	-5.0184	-1.6758	-.2804	-8.650	-5.149	-.565	-.423	.616	2.068
56	.6019	.3258	-4.8376	-1.6796	-.2733	-8.061	-5.155	-.523	-.444	.607	2.079
55	.6273	.3320	-4.6732	-1.6813	-.2669	-7.518	-5.151	-.483	-.465	.597	2.090
54	.6537	.3388	-4.5232	-1.6818	-.2609	-7.015	-5.139	-.444	-.485	.586	2.100
53	.6812	.3460	-4.3858	-1.6806	-.2554	-6.546	-5.120	-.406	-.505	.574	2.109
52	.7099	.3537	-4.2595	-1.6789	-.2503	-6.107	-5.096	-.369	-.524	.561	2.117
51	.7400	.3618	-4.1431	-1.6769	-.2455	-5.695	-5.067	-.333	-.543	.547	2.124
50	.7715	.3705	-4.0358	-1.6748	-.2410	-5.305	-5.033	-.298	-.561	.532	2.130
49	.8046	.3798	-3.9364	-1.6729	-.2368	-4.936	-4.996	-.262	-.579	.517	2.135
48	.8395	.3897	-3.8442	-1.6712	-.2328	-4.585	-4.955	-.227	-.596	.501	2.139
47	.8764	.4004	-3.7585	-1.6701	-.2291	-4.250	-4.910	-.191	-.613	.484	2.142
46	.9155	.4119	-3.6788	-1.6697	-.2256	-3.928	-4.863	-.156	-.629	.466	2.145
45	.9570	.4244	-3.6046	-1.6701	-.2223	-3.617	-4.812	-.120	-.644	.447	2.147
44	1.0013	.4378	-3.5354	-1.6714	-.2191	-3.317	-4.758	-.083	-.659	.427	2.148
43	1.0488	.4524	-3.4708	-1.6741	-.2161	-3.025	-4.702	-.045	-.673	.407	2.148
42	1.0997	.4683	-3.4104	-1.6779	-.2133	-2.740	-4.642	-.007	-.686	.386	2.147
41	1.1547	.4856	-3.3541	-1.6832	-.2106	-2.460	-4.578	.033	-.699	.364	2.145
40	1.2143	.5047	-3.3014	-1.6901	-.2081	-2.183	-4.511	.074	-.711	.341	2.143
39	1.2792	.5257	-3.2521	-1.6988	-.2057	-1.909	-4.440	.117	-.722	.317	2.140
38	1.3502	.5490	-3.2060	-1.7095	-.2034	-1.635	-4.365	.163	-.732	.293	2.136
37	1.4286	.5750	-3.1630	-1.7223	-.2012	-1.359	-4.285	.211	-.741	.268	2.131
36	1.5155	.6041	-3.1229	-1.7375	-.1991	-1.079	-4.199	.262	-.749	.242	2.125
35	1.6127	.6370	-3.0854	-1.7555	-.1972	-.794	-4.107	.318	-.756	.216	2.119
34	1.7223	.6745	-3.0505	-1.7764	-.1953	-.498	-4.009	.379	-.763	.189	2.112
33	1.8472	.7175	-3.0181	-1.8006	-.1935	-.190	-3.903	.446	-.769	.161	2.104
32	1.9911	.7675	-2.9880	-1.8284	-.1918	.136	-3.788	.522	-.774	.132	2.096
31	2.1591	.8264	-2.9602	-1.8604	-.1903	.486	-3.663	.607	-.778	.102	2.087
30	2.3584	.8968	-2.9346	-1.8970	-.1888	.869	-3.527	.705	-.780	.072	2.078
29	2.5993	.9825	-2.9111	-1.9388	-.1874	1.297	-3.376	.820	-.781	.041	2.068
28	2.8972	1.0890	-2.8898	-1.9867	-.1861	1.787	-3.210	.960	-.781	.009	2.058
27	3.2766	1.2256	-2.8706	-2.0414	-.1849	2.368	-3.025	1.133	-.780	-.024	2.048
26	3.7779	1.4068	-2.8536	-2.1039	-.1838	3.084	-2.818	1.357	-.778	-.058	2.039
25	4.4739	1.6597	-2.8387	-2.1758	-.1829	4.017	-2.586	1.661	-.775	-.093	2.030
24	5.5105	2.0376	-2.8260	-2.2584	-.1820	5.328	-2.322	2.105	-.771	-.129	2.022
23	7.227	2.665	-2.8157	-2.3539	-.1813	7.395	-2.020	2.826	-.765	-.166	2.015
22	10.645	3.918	-2.8080	-2.4649	-.1808	11.342	-1.670	4.254	-.758	-.205	2.009
21	20.861	7.670	-2.8034	-2.5946	-.1804	22.789	-1.264	8.410	-.749	-.246	2.005
20	∞	∞	-------	-------	-------	∞	-------	∞	-------	-.289	-------
					$\theta_{s_0} = 20^\circ$; $u_{s_0}/c = 0.50$; $M^0 = 1.65$						
44.4235	0.9198	0.2448	-9.1233	-1.3341	-0.9869	-23.512	-4.465	-3.346	-0.176	1.608	5.142
44	.9415	.2468	-8.1415	-1.8563	-.8791	-20.336	-6.306	-2.927	-.200	1.651	5.298
43	.9940	.2522	-6.6320	-2.3898	-.7082	-15.349	-8.322	-2.248	-.258	1.762	5.697
42	1.0490	.2585	-5.7033	-2.5384	-.6003	-12.166	-9.002	-1.801	-.320	1.877	6.113
41	1.1074	.2657	-5.0677	-2.5538	-.5253	-9.897	-9.182	-1.474	-.386	1.991	6.533
40	1.1699	.2739	-4.6026	-2.5184	-.4698	-8.156	-9.151	-1.219	-.455	2.105	6.953
39	1.2374	.2831	-4.2464	-2.4640	-.4271	-6.746	-9.016	-1.009	-.528	2.217	7.371
38	1.3108	.2935	-3.9644	-2.4045	-.3928	-5.557	-8.832	-.830	-.605	2.327	7.786
37	1.3912	.3053	-3.7354	-2.3464	-.3649	-4.518	-8.621	-.671	-.685	2.435	8.197
36	1.4802	.3187	-3.5459	-2.2926	-.3417	-3.584	-8.395	-.526	-.768	2.541	8.603
35	1.5792	.3340	-3.3868	-2.2448	-.3222	-2.721	-8.160	-.391	-.855	2.646	9.004
34	1.6907	.3515	-3.2515	-2.2037	-.3054	-1.902	-7.920	-.260	-.945	2.749	9.399
33	1.8173	.3718	-3.1354	-2.1697	-.2909	-1.104	-7.674	-.131	-1.038	2.851	9.787
32	1.9629	.3957	-3.0351	-2.1432	-.2784	-.301	-7.418	-.001	-1.134	2.951	10.167
31	2.1327	.4239	-2.9478	-2.1245	-.2675	.504	-7.150	.134	-1.233	3.050	10.539
30	2.3337	.4578	-2.8718	-2.1140	-.2578	1.360	-6.868	.278	-1.335	3.148	10.900
29	2.5764	.4991	-2.8054	-2.1123	-.2495	2.287	-6.570	.437	-1.440	3.246	11.250
28	2.8762	.5510	-2.7475	-2.1200	-.2421	3.326	-6.245	.617	-1.548	3.344	11.587
27	3.2575	.6175	-2.6971	-2.1382	-.2356	4.538	-5.893	.830	-1.658	3.443	11.908
26	3.7608	.7064	-2.6535	-2.1681	-.2300	6.013	-5.502	1.092	-1.771	3.543	12.211
25	4.4591	.8306	-2.6164	-2.2114	-.2253	7.922	-5.065	1.434	-1.886	3.645	12.493
24	5.4980	1.0168	-2.5853	-2.2705	-.2213	10.592	-4.568	1.917	-2.003	3.750	12.750
23	7.2175	1.3270	-2.5604	-2.3485	-.2181	14.783	-3.996	2.680	-2.122	3.860	12.976
22	10.6372	1.9470	-2.5418	-2.4496	-.2157	22.770	-3.328	4.137	-2.242	3.976	13.166
21	20.8538	3.8057	-2.5298	-2.5798	-.2141	45.879	-2.537	8.354	-2.361	4.101	13.317
20	∞	∞	-------	-------	-------	∞	-------	∞	-------	4.238	-------

NACA

TABLE 1.- COEFFICIENT FUNCTIONS F, G, AND H AND

VARIABLES ξ, η, AND ζ - Continued

θ (deg)	F_1	F_3	G_1	G_2	G_3	H_1	H_2	H_3	ξ	η	ζ
$\theta_{s_0} = 20^\circ$, $u_{s_0}/c = 0.65$; $M^o = 2.51$											
33.0452	1.7177	0.1963	-5.2914	-1.2739	-0.6912	-14.002	-6.1995	-3.0889	-0.293	2.166	8.736
33	1.7245	.1967	-5.2178	-1.3306	-.6826	-13.6145	-6.4793	-3.0358	-.296	2.171	8.763
32.5	1.8014	.2014	-4.5517	-1.7834	-.6012	-10.0189	-8.6989	-2.5288	-.331	2.237	9.092
32	1.8826	.2066	-4.0752	-2.0317	-.5399	-7.1628	-9.9337	-2.1289	-.367	2.308	9.440
31.5	1.9692	.2125	-3.7127	-2.1697	-.4918	-5.1203	-10.6193	-1.7993	-.404	2.381	9.795
31	2.0621	.2190	-3.4381	-2.2449	-.4531	-3.2917	-10.9748	-1.5180	-.442	2.456	10.151
30.5	2.1624	.2263	-3.2145	-2.2829	-.4214	-1.6978	-11.1328	-1.2709	-.482	2.531	10.507
30	2.2712	.2344	-3.0314	-2.2985	-.3948	-.2622	-11.1532	-1.0483	-.523	2.607	10.860
29.5	2.3902	.2435	-2.8788	-2.3006	-.3720	1.0690	-11.0812	-.8432	-.565	2.683	11.209
29	2.5210	.2537	-2.7496	-2.2950	-.3526	2.3367	-10.9485	-.6936	-.608	2.759	11.555
28.5	2.6657	.2652	-2.6391	-2.2853	-.3357	3.5734	-10.7649	-.4653	-.652	2.835	11.896
28	2.8271	.2781	-2.5436	-2.2740	-.3211	4.8082	-10.5501	-.2843	-.697	2.911	12.227
27.5	3.0085	.2929	-2.4604	-2.2630	-.3081	6.0674	-10.2986	-.1040	-.743	2.988	12.550
27	3.2144	.3099	-2.3876	-2.2534	-.2967	7.3803	-10.0256	.0790	-.790	3.065	12.864
26.5	3.4501	.3296	-2.3232	-2.2462	-.2867	8.7772	-9.7328	.2686	-.838	3.142	13.169
26	3.7233	.3527	-2.2663	-2.2424	-.2777	10.2951	-9.4147	.4687	-.886	3.220	13.464
25.5	4.0443	.3800	-2.2158	-2.2425	-.2697	11.9809	-9.0749	.6846	-.935	3.298	13.747
25	4.4272	.4128	-2.1710	-2.2472	-.2627	13.8971	-8.7100	.9231	-.985	3.377	14.017
24.5	4.8927	.4531	-2.1313	-2.2572	-.2566	16.1298	-8.3225	1.1932	-1.036	3.457	14.272
24	5.4718	.5036	-2.0960	-2.2731	-.2510	18.8074	-7.906	1.508	-1.087	3.538	14.512
23.5	6.2129	.5686	-2.0652	-2.2958	-.2463	22.127	-7.459	1.889	-1.139	3.621	14.734
23	7.1971	.6552	-2.0381	-2.3262	-.2422	26.417	-6.973	2.368	-1.191	3.706	14.936
22.5	8.5702	.7767	-2.0150	-2.3653	-.2387	32.264	-6.443	3.006	-1.243	3.793	15.115
22	10.624	.959	-1.9956	-2.4143	-.2358	40.833	-5.865	3.922	-1.295	3.882	15.267
21.5	14.036	1.263	-1.9800	-2.4748	-.2336	54.848	-5.229	5.392	-1.345	3.974	15.387
21	20.849	1.872	-1.9685	-2.5488	-.2319	82.465	-4.524	8.243	-1.392	4.070	15.465
20.5	41.255	3.699	-1.9611	-2.6387	-.2308	164.428	-3.732	16.605	-1.431	4.170	15.470
20	∞	∞	-------	-------	-------	∞	-------	∞	------	4.275	------
$\theta_{s_0} = 20^\circ$; $u_{s_0}/c = 0.75$; $M^o = 3.36$											
28.1798	2.6738	0.1536	-3.2411	-1.2036	-0.4740	-2.4775	-9.1430	-2.6656	-0.397	2.381	13.058
28	2.7375	.1559	-3.0346	-1.3811	-.4526	-.3678	-10.4394	-2.4402	-.410	2.405	13.231
27.5	2.9279	.1632	-2.6556	-1.7169	-.4042	4.7440	-12.9466	-1.8962	-.446	2.478	13.744
27	3.1414	.1717	-2.2766	-1.9100	-.3672	9.1888	-14.4089	-1.4364	-.482	2.555	14.277
26.5	3.3840	.1818	-2.0433	-2.0243	-.3381	13.321	-15.230	-1.028	-.519	2.635	14.819
26	3.6634	.1936	-1.8641	-2.0936	-.3146	17.402	-15.637	-.650	-.557	2.717	15.363
25.5	3.9900	.2079	-1.7227	-2.1372	-.2953	21.628	-15.751	-.285	-.596	2.801	15.903
25	4.3782	.2251	-1.6088	-2.1668	-.2794	26.194	-15.646	.081	-.636	2.886	16.435
24.5	4.8487	.2463	-1.5157	-2.1899	-.2662	31.325	-15.370	.462	-.677	2.972	16.957
24	5.4326	.2729	-1.4385	-2.2117	-.2550	37.331	-14.944	.876	-.719	3.060	17.464
23.5	6.1785	.3075	-1.3743	-2.2361	-.2458	44.631	-14.380	1.346	-.761	3.150	17.955
23	7.1673	.3535	-1.3205	-2.2663	-.2371	53.970	-13.691	1.908	-.804	3.242	18.425
22.5	8.5449	.4184	-1.2757	-2.3051	-.2315	66.552	-12.860	2.625	-.847	3.336	18.872
22	10.6027	.5159	-1.2391	-2.3555	-.2264	84.874	-11.879	3.616	-.890	3.433	19.293
21.5	14.020	.679	-1.2100	-2.4207	-.2225	114.676	-10.734	5.159	-.933	3.534	19.686
21	20.837	1.005	-1.1884	-2.5043	-.2196	173.262	-9.399	8.084	-.974	3.639	20.053
20.5	41.246	1.985	-1.1746	-2.6112	-.2178	346.218	-7.836	16.522	-1.010	3.750	20.410
20	∞	∞	-------	-------	-------	∞	-------	∞	------	3.869	------
$\theta_{s_0} = 20^\circ$; $u_{s_0}/c = 0.85$; $M^o = 5.54$											
24.433	4.7991	0.1014	-0.5386	-1.1540	-0.2755	59.37	-19.17	-1.82	-0.517	2.471	17.529
24.0	5.3274	.1102	-.1718	-1.5183	-.2496	78.62	-23.90	-1.04	-.535	2.531	18.403
23.5	6.0890	.1232	.1101	-1.7753	-.2273	101.29	-26.83	-.20	-.556	2.607	19.505
23.0	7.0920	.1409	.3019	-1.9406	-.2106	127.82	-28.10	.67	-.576	2.689	20.666
22.5	8.4827	.1660	.4381	-2.0612	-.1980	162.06	-28.19	1.64	-.595	2.777	21.863
22.0	10.5528	.2039	.5374	-2.1647	-.1885	210.42	-27.34	2.85	-.613	2.869	23.087
21.5	13.982	.267	.6103	-2.2698	-.1814	287.84	-25.68	4.59	-.628	2.967	24.338
21.25	16.717	.318	.6384	-2.3276	-.1788	348.36	-24.51	5.87	-.634	3.018	24.978
21.0	20.811	.395	.6620	-2.3915	-.1766	438.11	-23.15	7.70	-.638	3.071	25.635
20.75	27.624	.523	.6807	-2.4633	-.1749	586.32	-21.54	10.64	-.640	3.126	26.322
20.5	41.234	.779	.6945	-2.5449	-.1737	880.34	-19.73	16.33	-.638	3.183	27.069
20.25	82.030	1.549	.7033	-2.6388	-.1729	1758.02	-17.67	33.03	-.628	3.242	27.967
20	∞	-------	-------	-------	-------	∞	-------	∞	------	3.303	------

NACA

TABLE 1.- COEFFICIENT FUNCTIONS F, G, AND H AND

VARIABLES ξ, η, AND ζ - Continued

θ (deg)	F_1	F_3	G_1	G_2	G_3	H_1	H_2	H_3	ξ	η	ζ
$\theta_{s_0} = 30^\circ$; $u_{s_0}/c = 0.35$; $M^\circ = 1.515$											
63.3199	0.5756	0.2398	-5.6588	-0.4028	-0.2647	-13.6739	-1.6360	-0.9703	-0.381	0.877	1.686
63	.5846	.2415	-5.5395	-.4613	-.2595	-13.2030	-1.8696	-.9359	-.388	.869	1.674
62	.6132	.2472	-5.2102	-.6100	-.2453	-11.8888	-2.4651	-.8381	-.410	.847	1.645
61	.6431	.2531	-4.9341	-.7203	-.2334	-10.7725	-2.9077	-.7523	-.431	.826	1.624
60	.6743	.2595	-4.6988	-.8040	-.2233	-9.7876	-3.2422	-.6755	-.451	.806	1.609
59	.7069	.2663	-4.4955	-.8688	-.2146	-8.9229	-3.4972	-.6054	-.471	.787	1.599
58	.7412	.2737	-4.3179	-.9197	-.2069	-8.1487	-3.6927	-.5405	-.490	.768	1.592
57	.7774	.2814	-4.1614	-.9603	-.2002	-7.4469	-3.8422	-.4797	-.508	.749	1.588
56	.8156	.2899	-4.0224	-.9932	-.1941	-6.8055	-3.9555	-.4219	-.525	.730	1.587
55	.8562	.2989	-3.8981	-1.0203	-.1887	-6.2120	-4.0388	-.3664	-.541	.711	1.587
54	.8994	.3088	-3.7864	-1.0429	-.1839	-5.6591	-4.0978	-.3125	-.557	.692	1.589
53	.9456	.3196	-3.6855	-1.0623	-.1794	-5.1399	-4.1370	-.2598	-.572	.673	1.593
52	.9953	.3312	-3.5942	-1.0792	-.1755	-4.6477	-4.1585	-.2076	-.586	.654	1.598
51	1.0488	.3440	-3.5110	-1.0945	-.1718	-4.1783	-4.1649	-.1554	-.599	.634	1.604
50	1.1068	.3581	-3.4352	-1.1082	-.1684	-3.7265	-4.1576	-.1029	-.611	.614	1.611
49	1.1700	.3736	-3.3659	-1.1213	-.1652	-3.2884	-4.1381	-.0494	-.623	.594	1.619
48	1.2392	.3908	-3.3024	-1.1342	-.1623	-2.8600	-4.1067	.0055	-.634	.574	1.628
47	1.3156	.4099	-3.2444	-1.1470	-.1596	-2.4374	-4.0645	.0625	-.644	.553	1.638
46	1.4004	.4315	-3.1911	-1.1602	-.1573	-2.0170	-4.0115	.1223	-.653	.532	1.648
45	1.4953	.4560	-3.1422	-1.1741	-.1549	-1.5944	-3.9481	.1856	-.661	.511	1.659
44	1.6025	.4839	-3.0974	-1.1890	-.1528	-1.1646	-3.8742	.2535	-.668	.490	1.671
43	1.7247	.5160	-3.0564	-1.2052	-.1509	-.7226	-3.7885	.3273	-.674	.468	1.684
42	1.8657	.5536	-3.0189	-1.2229	-.1492	-.2612	-3.6912	.4085	-.679	.446	1.697
41	2.0307	.5976	-2.9847	-1.2426	-.1475	.2282	-3.5828	.4995	-.683	.424	1.711
40	2.2268	.6506	-2.9536	-1.2645	-.1459	.7570	-3.4605	.6032	-.686	.402	1.726
39	2.4643	.7152	-2.9256	-1.2891	-.1446	1.3412	-3.3236	.7241	-.687	.380	1.742
38	2.7586	.7957	-2.9005	-1.3167	-.1433	2.0040	-3.1706	.8684	-.687	.358	1.759
37	3.1340	.8991	-2.8782	-1.3479	-.1422	2.7815	-2.9995	1.0462	-.686	.336	1.777
36	3.6310	1.0366	-2.8586	-1.3833	-.1412	3.7334	-2.8084	1.2742	-.683	.314	1.796
35	4.322	1.229	-2.8417	-1.4235	-.1403	4.9658	-2.5942	1.5820	-.678	.292	1.817
34	5.354	1.517	-2.8277	-1.4693	-.1395	6.6894	-2.3534	2.0291	-.671	.270	1.840
33	7.066	1.997	-2.8165	-1.5217	-.1390	9.3956	-2.0820	2.7534	-.662	.248	1.866
32	10.477	2.954	-2.8083	-1.5819	-.1385	14.5498	-1.7744	4.1682	-.649	.227	1.897
31	20.686	5.827	-2.8031	-1.6514	-.1383	29.4903	-1.4246	8.3384	-.629	.207	1.937
30	∞	∞	-------	-------	-------	∞	-------	∞	------	.188	------
$\theta_{s_0} = 30^\circ$; $u_{s_0}/c = 0.55$; $M^\circ = 2.33$											
44.1237	1.5139	0.1919	-4.1222	-0.6533	-0.3194	-9.4820	-4.0531	-1.4287	-0.519	1.669	7.321
44	1.5289	.1930	-4.0623	-.6959	-.3140	-9.0885	-4.3020	-1.3811	-.525	1.671	7.345
43	1.6584	.2032	-3.6648	-.9447	-.2781	-6.2917	-5.7816	-1.0407	-.573	1.697	7.580
42	1.8057	.2155	-3.3764	-1.0869	-.2514	-3.9464	-6.6285	-.7551	-.622	1.726	7.834
41	1.9764	.2303	-3.1676	-1.1730	-.2309	-1.8409	-7.1031	-.5003	-.671	1.757	8.098
40	2.1778	.2484	-2.9873	-1.2281	-.2151	.1649	-7.3365	-.2608	-.720	1.789	8.366
39	2.4202	.2708	-2.8511	-1.2664	-.2016	2.1860	-7.4001	-.0240	-.769	1.821	8.635
38	2.7192	.2990	-2.7409	-1.2965	-.1910	4.3070	-7.3343	.2219	-.818	1.853	8.901
37	3.0992	.3355	-2.6508	-1.3238	-.1822	6.6792	-7.1600	.4910	-.867	1.886	9.162
36	3.6007	.3846	-2.5771	-1.3522	-.1751	9.4778	-6.8870	.8028	-.916	1.919	9.416
35	4.2968	.4536	-2.5172	-1.3850	-.1693	13.012	-6.517	1.190	-.964	1.952	9.662
34	5.3330	.5574	-2.4692	-1.4252	-.1646	17.878	-6.046	1.713	-1.011	1.986	9.897
33	7.0493	.7309	-2.4210	-1.4759	-.1610	25.413	-5.462	2.512	-1.057	2.021	10.120
32	10.465	1.079	-2.4052	-1.5406	-.1585	39.674	-4.748	4.003	-1.099	2.058	10.333
31	20.678	2.125	-2.3885	-1.6240	-.1569	81.244	-3.873	8.253	-1.134	2.097	10.550
30	∞	∞	-------	-------	-------	∞	-------	∞	------	2.139	------

NACA

TABLE 1.- COEFFICIENT FUNCTIONS F, G, AND H AND

VARIABLES ξ, η, AND ζ - Continued

θ (deg)	F_1	F_3	G_1	G_2	G_3	H_1	H_2	H_3	ξ	η	ζ
$\theta_{s_0} = 30^\circ$; $u_{s_0}/c = 0.65$; $M^o = 3.16$											
39.1748	2.3176	0.1561	-3.1456	-0.6787	-0.2556	-1.983	-6.428	-1.248	-0.644	1.913	9.913
39.0	2.3652	.1584	-3.0704	-.7418	-.2494	-.984	-6.941	-1.153	-.652	1.919	9.985
38.75	2.4361	.1618	-2.9725	-.8202	-.2412	.398	-7.569	-1.022	-.663	1.928	10.093
38.5	2.5107	.1655	-2.8844	-.8871	-.2336	1.738	-8.104	-.897	-.674	1.937	10.205
38.25	2.5894	.1694	-2.8046	-.9445	-.2268	3.049	-8.556	-.776	-.685	1.947	10.320
38.0	2.6725	.1737	-2.7324	-.9940	-.2204	4.341	-8.935	-.658	-.696	1.957	10.438
37.75	2.7606	.1781	-2.6663	-1.0370	-.2154	5.623	-9.259	-.543	-.707	1.967	10.559
37.5	2.8540	.1830	-2.6060	-1.0746	-.2093	6.905	-9.530	-.430	-.718	1.978	10.682
37.25	2.9535	.1881	-2.5507	-1.1077	-.2043	8.195	-9.747	-.319	-.729	1.989	10.807
37.0	3.0596	.1937	-2.4996	-1.1370	-.1997	9.503	-9.931	-.207	-.740	2.000	10.934
36.75	3.1731	.1997	-2.4527	-1.1631	-.1954	10.836	-10.070	-.096	-.751	2.011	11.062
36.50	3.2949	.2062	-2.4094	-1.1867	-.1914	12.203	-10.177	.016	-.761	2.023	11.191
36.25	3.4260	.2132	-2.3692	-1.2080	-.1878	13.614	-10.256	.129	-.771	2.035	11.321
36.0	3.5675	.2209	-2.3320	-1.2276	-.1843	15.082	-10.356	.246	-.781	2.047	11.452
35.75	3.7208	.2293	-2.2974	-1.2458	-.1811	16.615	-10.352	.365	-.791	2.059	11.584
35.50	3.8875	.2384	-2.2654	-1.2629	-.1782	18.227	-10.327	.487	-.801	2.071	11.716
35.25	4.0695	.2484	-2.2355	-1.2790	-.1754	19.935	-10.302	.615	-.811	2.084	11.849
35.0	4.2693	.2595	-2.2078	-1.2945	-.1728	21.755	-10.256	.748	-.821	2.097	11.981
34.75	4.4894	.2717	-2.1820	-1.3096	-.1703	23.708	-10.191	.889	-.831	2.110	12.117
34.50	4.7334	.2854	-2.1580	-1.3244	-.1680	25.817	-10.101	1.038	-.841	2.123	12.247
34.25	5.0055	.3007	-2.1356	-1.3392	-.1660	28.116	-9.995	1.198	-.851	2.136	12.380
34.0	5.3110	.3179	-2.1149	-1.3541	-.1640	30.643	-9.867	1.370	-.861	2.150	12.513
33.75	5.6564	.3374	-2.0957	-1.3693	-.1622	33.441	-9.722	1.560	-.870	2.164	12.646
33.50	6.0503	.3598	-2.0779	-1.3849	-.1606	36.577	-9.557	1.766	-.879	2.178	12.779
33.25	6.5041	.3853	-2.0614	-1.4010	-.1589	40.149	-9.384	1.997	-.888	2.192	12.912
33.0	7.0325	.4158	-2.0464	-1.4178	-.1576	44.188	-9.168	2.259	-.897	2.206	13.045
32.75	7.6561	.4515	-2.0347	-1.4355	-.1563	48.916	-8.945	2.558	-.905	2.221	13.178
32.50	8.4033	.4944	-2.0198	-1.4541	-.1550	54.509	-8.703	2.908	-.913	2.236	13.311
32.25	9.3154	.5469	-2.0085	-1.4739	-.1540	61.244	-8.439	3.325	-.921	2.251	13.445
32.0	10.454	.613	-1.9982	-1.4950	-.1530	69.56	-8.15	3.83	-.928	2.267	13.580
31.75	11.916	.697	-1.9891	-1.5176	-.1522	80.16	-7.84	4.47	-.934	2.283	13.716
31.50	13.864	.810	-1.9812	-1.5417	-.1516	94.09	-7.51	5.31	-.939	2.299	13.855
31.25	16.588	.967	-1.9745	-1.5677	-.1509	113.46	-7.16	6.46	-.943	2.316	13.997
31.0	20.672	1.204	-1.9688	-1.5956	-.1505	142.31	-6.78	8.17	-.946	2.333	14.145
30.75	27.474	1.599	-1.9644	-1.6258	-.1500	190.10	-6.37	10.97	-.947	2.351	14.304
30.5	41.072	2.389	-1.9612	-1.6584	-.1498	285.20	-5.93	16.51	-.944	2.369	14.482
30.25	81.852	4.760	-1.9593	-1.6938	-.1496	569.68	-5.46	33.10	-.935	2.388	14.702
30.0	∞	∞	-------	-------	------	∞	------	∞	------	2.408	------

NACA

TABLE 1.- COEFFICIENT FUNCTIONS F, G, AND H AND

VARIABLES ξ, η, AND ζ - Concluded

θ (deg)	F_1	F_3	G_1	G_2	G_3	H_1	H_2	H_3	ξ	η	ζ
$\theta_{s_0} = 30^\circ$; $u_{s_0}/c = 0.70$; $M^0 = 3.85$											
37.2678	2.9101	0.1365	-2.6024	-0.6856	-0.2198	7.3361	-8.5699	-1.0607	-0.712	2.009	11.186
37.25	2.9175	.1367	-2.5948	-.6927	-.2192	7.4975	-8.6338	-1.0485	-.713	2.010	11.197
37	3.0255	.1405	-2.4950	-.7837	-.2116	9.7280	-9.4811	-.8814	-.722	2.019	11.351
36.75	3.1408	.1448	-2.4062	-.8608	-.2048	11.913	-10.167	-.720	-.731	2.029	11.512
36.5	3.2642	.1494	-2.3271	-.9265	-.1986	14.078	-10.728	-.562	-.740	2.039	11.679
36.25	3.3968	.1543	-2.2561	-.9831	-.1929	16.245	-11.190	-.407	-.748	2.050	11.851
36	3.5399	.1596	-2.1921	-1.0321	-.1877	18.438	-11.550	-.254	-.756	2.062	12.027
35.75	3.6946	.1655	-2.1342	-1.0750	-.1830	20.678	-11.841	-.100	-.764	2.074	12.207
35.5	3.8627	.1719	-2.0818	-1.1129	-.1786	22.990	-12.053	.054	-.772	2.086	12.390
35.25	4.0461	.1790	-2.0341	-1.1467	-.1747	25.396	-12.213	.212	-.780	2.099	12.575
35	4.2471	.1869	-1.9907	-1.1771	-.1710	27.923	-12.318	.373	-.788	2.112	12.762
34.75	4.4686	.1956	-1.9509	-1.2049	-.1676	30.603	-12.365	.539	-.796	2.126	12.951
34.5	4.7138	.2052	-1.9146	-1.2306	-.1646	33.472	-12.373	.713	-.803	2.140	13.142
34.25	4.9871	.2160	-1.8813	-1.2547	-.1617	36.570	-12.338	.896	-.810	2.154	13.334
34	5.2937	.2283	-1.8509	-1.2776	-.1591	39.951	-12.271	1.090	-.817	2.169	13.527
33.75	5.6403	.2422	-1.8230	-1.2997	-.1567	43.679	-12.164	1.299	-.824	2.184	13.722
33.5	6.0354	.2581	-1.7975	-1.3213	-.1544	47.832	-12.023	1.527	-.830	2.199	13.917
33.25	6.4902	.2765	-1.7742	-1.3427	-.1525	52.517	-11.853	1.777	-.836	2.215	14.113
33	7.0198	.2979	-1.7530	-1.3643	-.1506	57.872	-11.645	2.057	-.842	2.231	14.310
32.75	7.6444	.3234	-1.7336	-1.3863	-.1489	64.085	-11.409	2.374	-.847	2.247	14.508
32.5	8.3925	.3540	-1.7161	-1.4090	-.1474	71.419	-11.138	2.742	-.852	2.264	14.707
32.25	9.3056	.3915	-1.7004	-1.4325	-.1461	80.252	-10.839	3.175	-.856	2.281	14.907
32	10.445	.438	-1.6863	-1.4573	-.1448	91.152	-10.506	3.701	-.859	2.299	15.109
31.75	11.909	.499	-1.6739	-1.4835	-.1438	105.002	-10.136	4.357	-.861	2.317	15.314
31.5	13.858	.579	-1.6628	-1.5114	-.1427	123.285	-9.738	5.211	-.862	2.336	15.522
31.25	16.593	.692	-1.6531	-1.5414	-.1417	148.656	-9.301	6.379	-.861	2.355	15.735
31	20.668	.861	-1.6460	-1.5735	-.1416	186.431	-8.828	8.099	-.858	2.375	15.956
30.75	27.473	1.143	-1.6406	-1.6083	-.1405	249.018	-8.310	10.921	-.852	2.395	16.189
30.5	41.074	1.708	-1.6357	-1.6461	-.1405	373.615	-7.751	16.496	-.841	2.416	16.443
30.25	81.862	3.402	-1.6331	-1.6872	-.1404	746.260	-7.148	33.084	-.820	2.438	16.743
30	∞	∞	-------	-------	-------	∞	-------	∞	------	2.461	------
$\theta_{s_0} = 30^\circ$; $u_{s_0}/c = 0.75$; $M^0 = 5.011$											
35.6310	3.7333	0.1158	-1.9555	-0.6906	-0.1830	27.724	-12.093	-0.7734	-0.782	2.090	12.060
35.5	3.8244	.1184	-1.9027	-.7446	-.1801	29.898	-12.720	-.6579	-.785	2.095	12.191
35.25	4.0102	.1237	-1.8119	-.8349	-.1749	34.082	-13.732	-.4409	-.790	2.105	12.451
35.0	4.2134	.1295	-1.7323	-.9116	-.1702	38.352	-14.537	-.2260	-.795	2.116	12.725
34.75	4.4369	.1359	-1.6620	-.9774	-.1660	42.773	-15.168	-.0105	-.799	2.128	13.010
34.5	4.6841	.1431	-1.5997	-1.0345	-.1622	47.416	-15.651	.2078	-.803	2.141	13.305
34.25	4.9593	.1511	-1.5442	-1.0846	-.1587	52.349	-16.005	.4315	-.807	2.155	13.610
34	5.2677	.1600	-1.4947	-1.1292	-.1556	57.665	-16.247	.6641	-.810	2.170	13.923
33.75	5.6161	.1702	-1.4505	-1.1695	-.1527	63.458	-16.390	.9084	-.813	2.185	14.244
33.5	6.0129	.1818	-1.4108	-1.2065	-.1502	69.861	-16.443	1.1689	-.816	2.201	14.573
33.25	6.4695	.1951	-1.3751	-1.2410	-.1478	77.030	-16.415	1.4505	-.818	2.217	14.909
33	7.0006	.2107	-1.3432	-1.2738	-.1458	85.140	-16.303	1.7597	-.820	2.234	15.252
32.75	7.6269	.2292	-1.3145	-1.3055	-.1438	94.595	-16.135	2.1054	-.821	2.252	15.602
32.5	8.3768	.2513	-1.2889	-1.3367	-.1422	105.67	-15.892	2.4994	-.822	2.270	15.958
32.25	9.2914	.2783	-1.2662	-1.3679	-.1407	118.97	-15.582	2.9592	-.822	2.289	16.325
32	10.4326	.3120	-1.2460	-1.3996	-.1393	135.02	-15.208	3.5095	-.821	2.308	16.701
31.75	11.8975	.3554	-1.2283	-1.4324	-.1382	156.11	-14.768	4.1905	-.819	2.328	17.089
31.5	13.8478	.4132	-1.2130	-1.4666	-.1372	183.48	-14.263	5.0679	-.816	2.349	17.493
31.25	16.5752	.4932	-1.2001	-1.5028	-.1363	221.44	-13.691	6.2601	-.810	2.371	17.918
31	20.662	.615	-1.1894	-1.5415	-.1356	277.91	-13.048	8.004	-.802	2.393	18.369
30.75	27.469	.818	-1.1810	-1.5832	-.1351	371.43	-12.332	10.849	-.791	2.416	18.860
30.5	41.072	1.222	-1.1748	-1.6284	-.1347	557.51	-11.538	16.449	-.773	2.440	19.421
30.25	81.858	2.435	-1.1711	-1.6778	-.1344	1113.8	-10.660	33.059	-.744	2.465	20.125
30	∞	∞	-------	-------	-------	∞	-------	∞	------	2.491	------

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TABLE 2.- R_{w_o}/R_{s_o} AND M^o AT VARIOUS

VALUES OF θ_{s_o}

$\theta_{s_o} = 10^o$		$\theta_{s_o} = 20^o$		$\theta_{s_o} = 30^o$	
M^o	R_{w_o}/R_{s_o}	M^o	R_{w_o}/R_{s_o}	M^o	R_{w_o}/R_{s_o}
1.09	7.63	1.22	-0.275	1.52	0.179
1.60	40.1	1.65	2.83	2.33	1.30
2.39	12.0	2.51	2.19	3.16	1.24
3.30	5.23	3.37	1.72	3.85	1.17
5.42	2.35	5.55	1.30	5.01	1.11

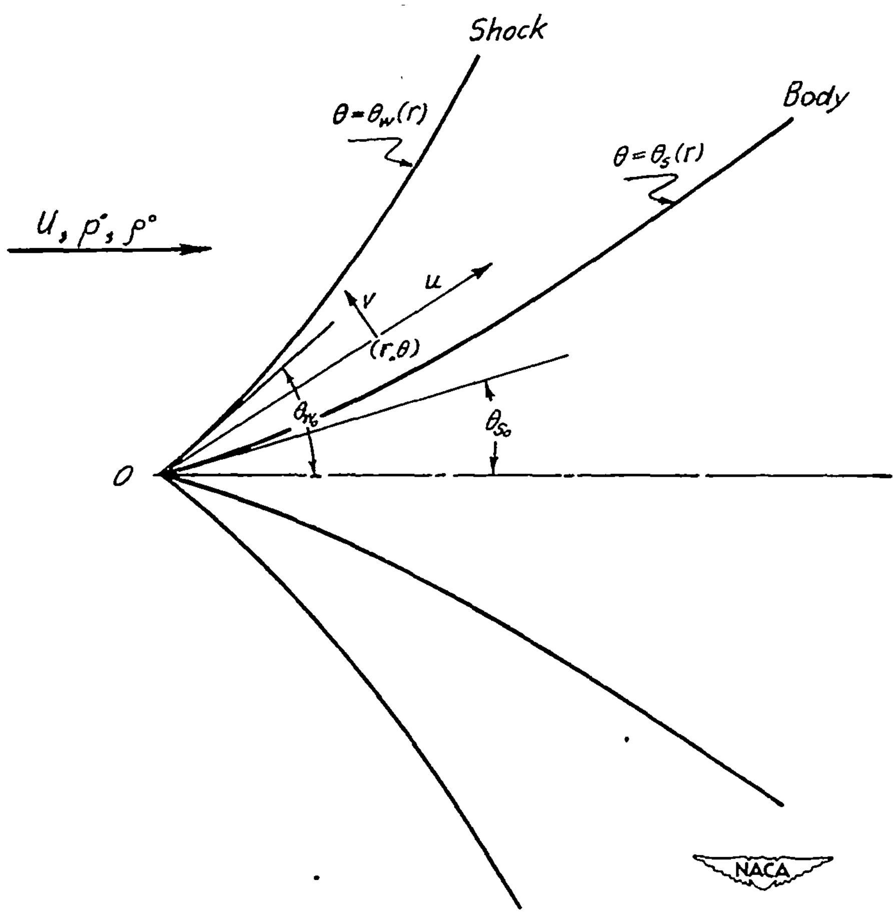

Figure 1.- General notations.

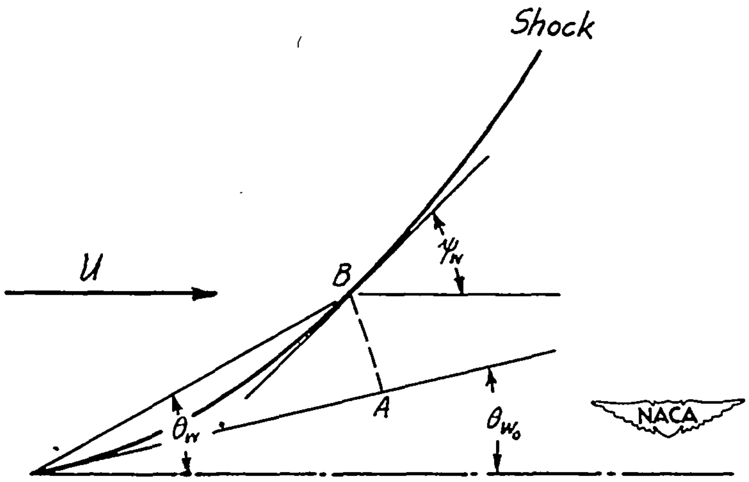

Figure 2.- Diagram illustrating formulation of boundary conditions at shock wave.

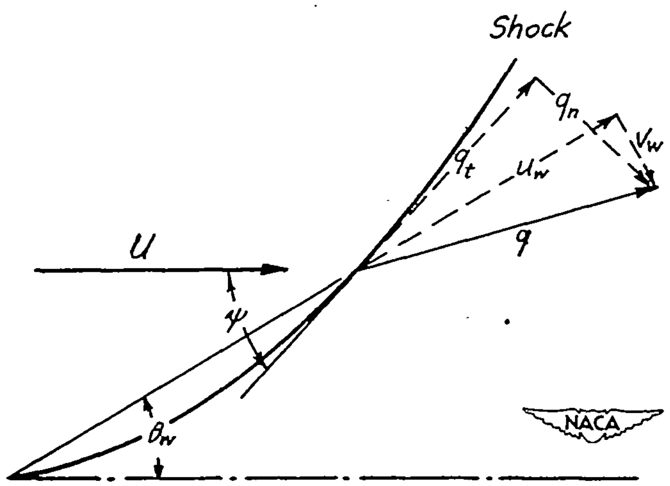

Figure 3.- Velocity components at a point on shock.

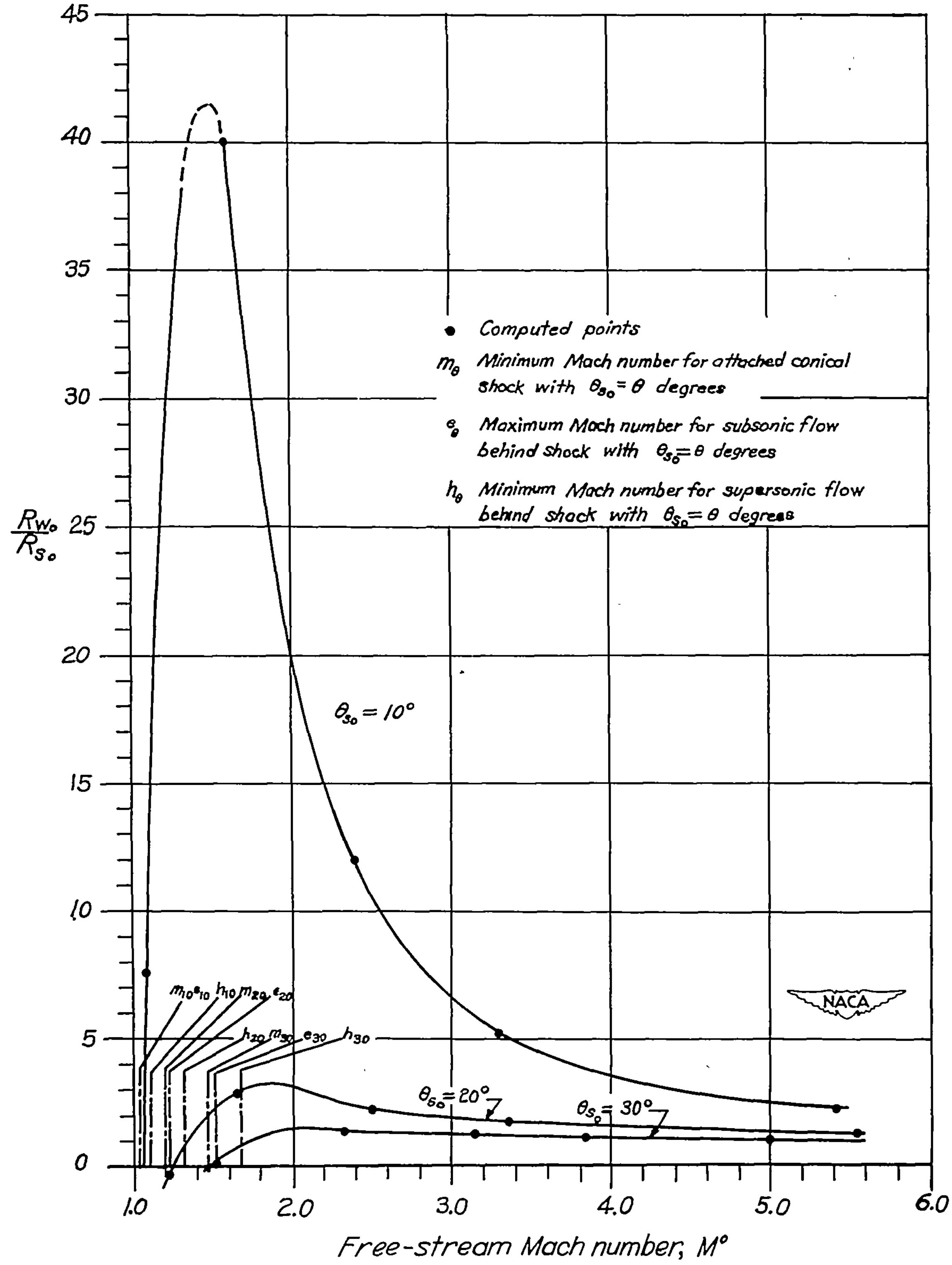

Figure 4.- Variation of ratio of curvature with M^o. Data for m_θ, ρ_θ, and h_θ taken from reference 7.

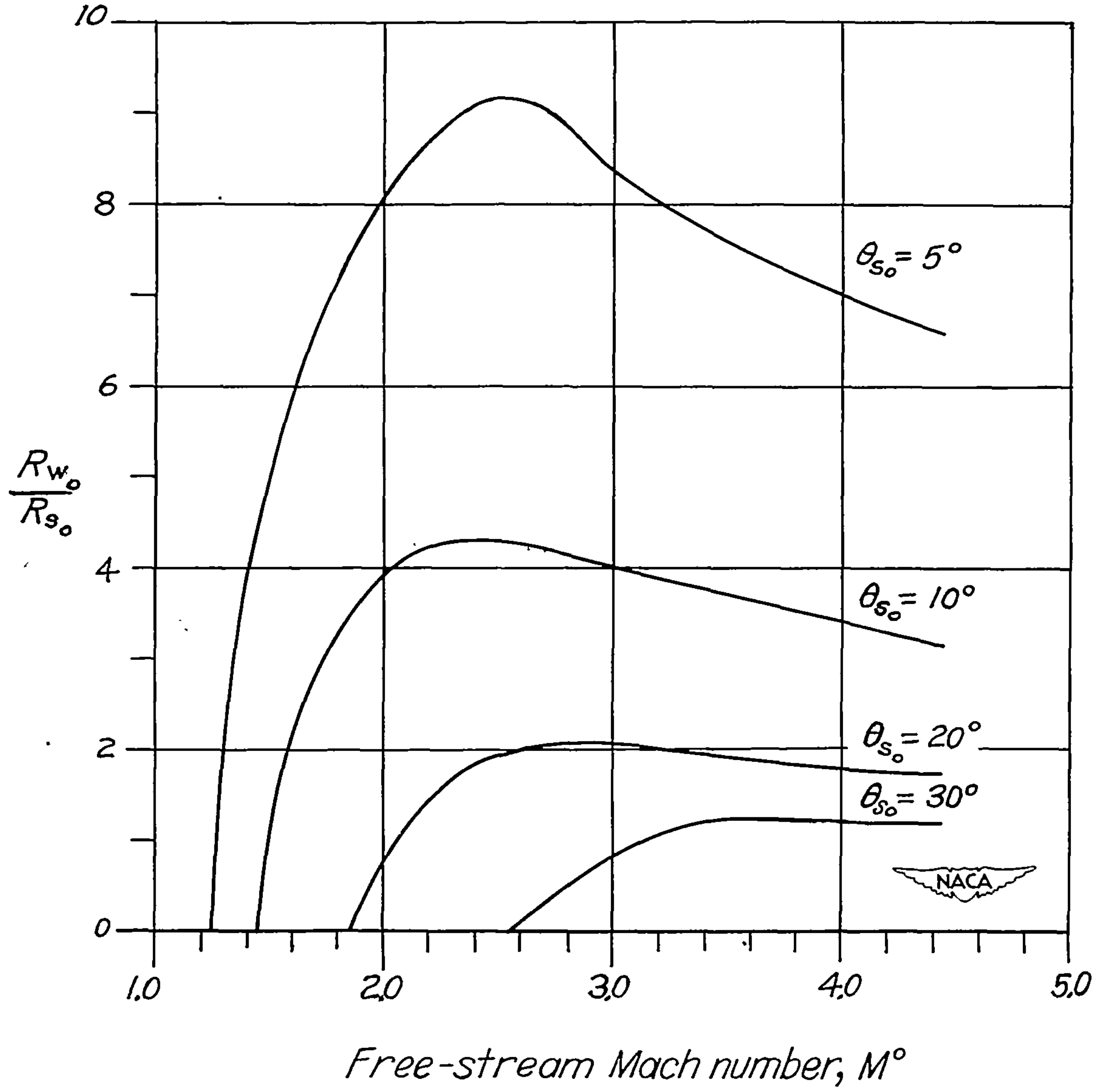

Figure 5.- Variation of ratio of curvature with M^o in two-dimensional case.